Interplay of Dynamics with Irregular Structure and Boundary in models of Self Organized Criticality

1 Introduction

We have done experiments (simulations) of various models of self organized criticality on Sierpinski Gasket which is a fractal of dimension 1.58. This object has three boundary points. We describe below the models and the results.

2 Models

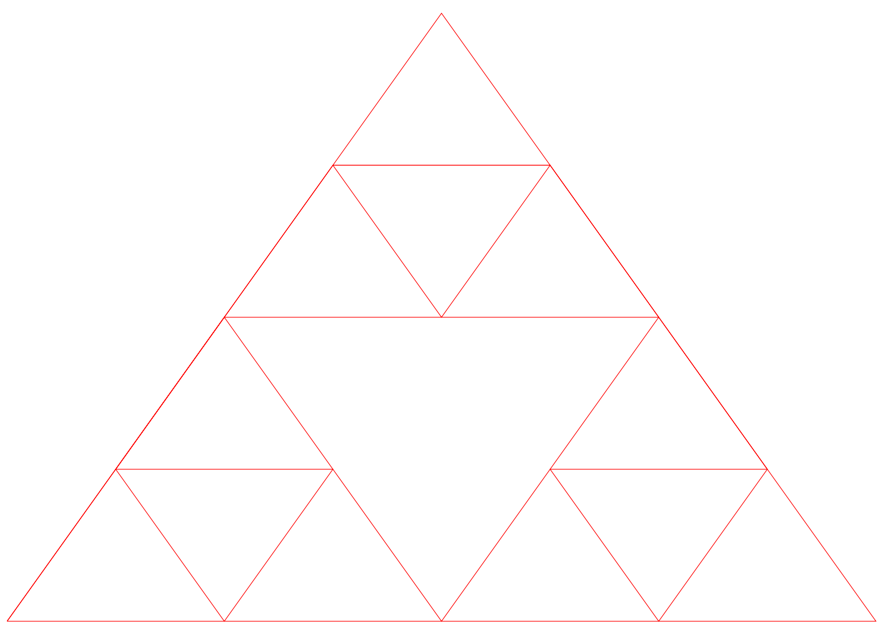


Figure 1: Sierpinski gasket of second order.

We simulate the sand-pile model and the Manna models on Sierpinski gasket. The Sierpinski gasket is a fractal of dimension 1.58. The number of sites in the gasket is given as [cite reference] . where S_0 is the number of sites in the zeroth order of the gasket, N is the order of the gasket. In Figure **??** we show a second

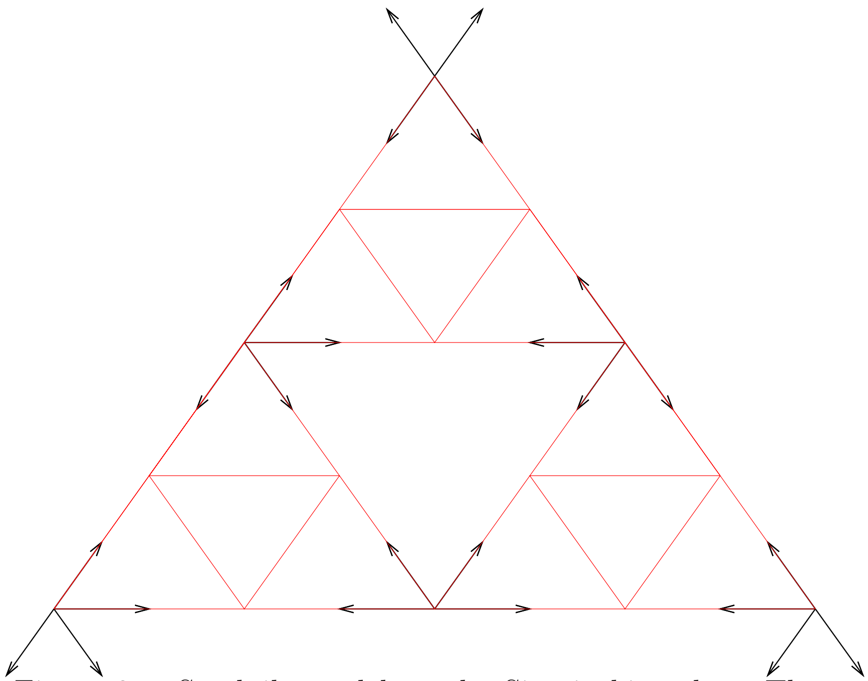


Figure 2: Sandpile model on the Sierpinski gasket. The arrows denote the four neighbours of a site.

order Sierpinski gasket. Each site in the system is connected to four neighbouring sites. The number of boundary points is 3 for all system sizes.

2.1 Sand-pile model

One of the site is randomly selected and a grain is added to the site. Addition of a grain increases the height at the site by 1. In this model we define the critical height z_c of any site in the system $z_c = 4$. If the height (z) is greater than or equal to z_c the site topples and the four grains are randomly distributed among the the four neighbours. During this step (defined as the micro time step t number of sites may become critical $(z = z_c)$. The toppling follows a parallel update method [cite Dhar Pradhan paper] A list of all unstable sites at time t is made, choose four grains from each site in this list and assign them randomly to the four neighbouring sites. All the grains which are assined to various sites are added to the sites at the same time.

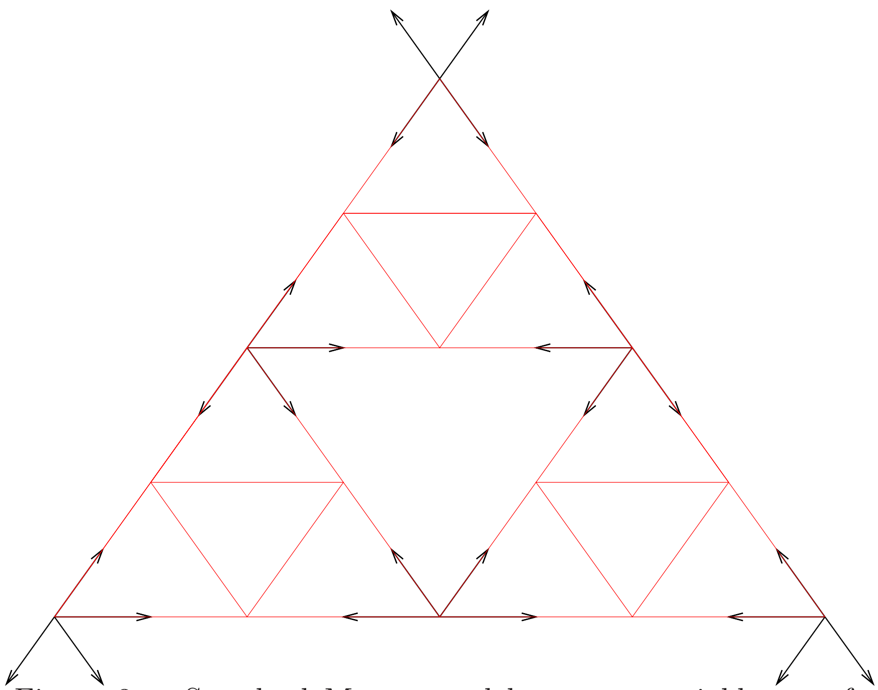


Figure 3: Standard Manna model, any two neighbours of the four denoted by arrows are randomly selected to dissipate the two grains after a site topples. In Manna models the critical height is 2.

2.2 Manna model

For Manna model (MM), $Z_c = 2$. If $Z_i \geq 2$ the site topples and two grains are assigned to any two randomly selected neighbours. To incorporate anisotropies in the dynamics we consider variations of the MM.

2.2.1 Manna model type A

In the type A Manna model (MMA), $Z_c = 2$. In this model we pair the two opposite sites.

3 Results

We present the results of the simulations for various models.

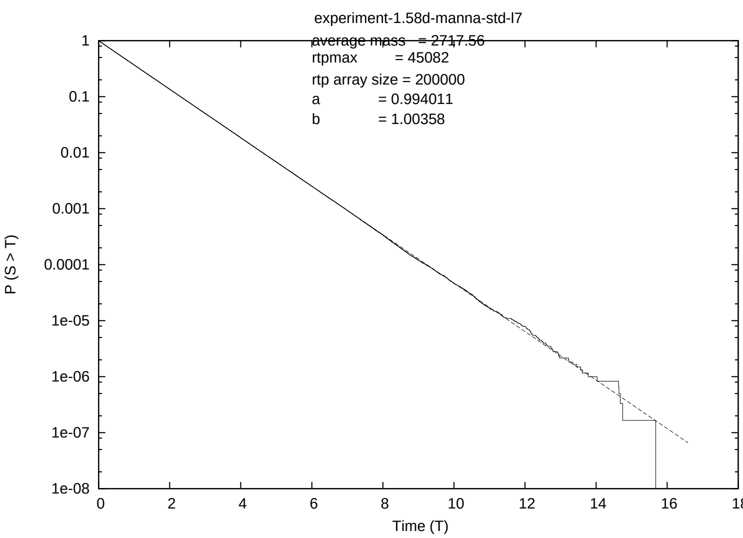
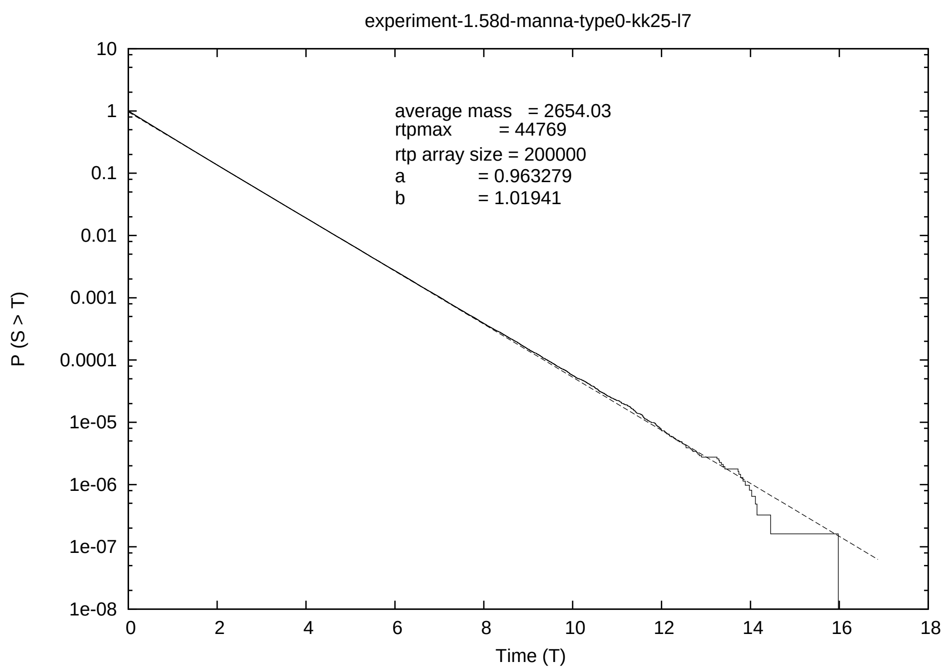


Figure 4:



experiment-1.58d-manna-type0-kk25-l7
average mass = 2654.03
rtpmax = 44769
rtp array size = 200000
a = 0.963279
b = 1.01941
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
P (S > T)
0
2
4
6
8
10
12
14
16
18
Time (T)

Figure 5:

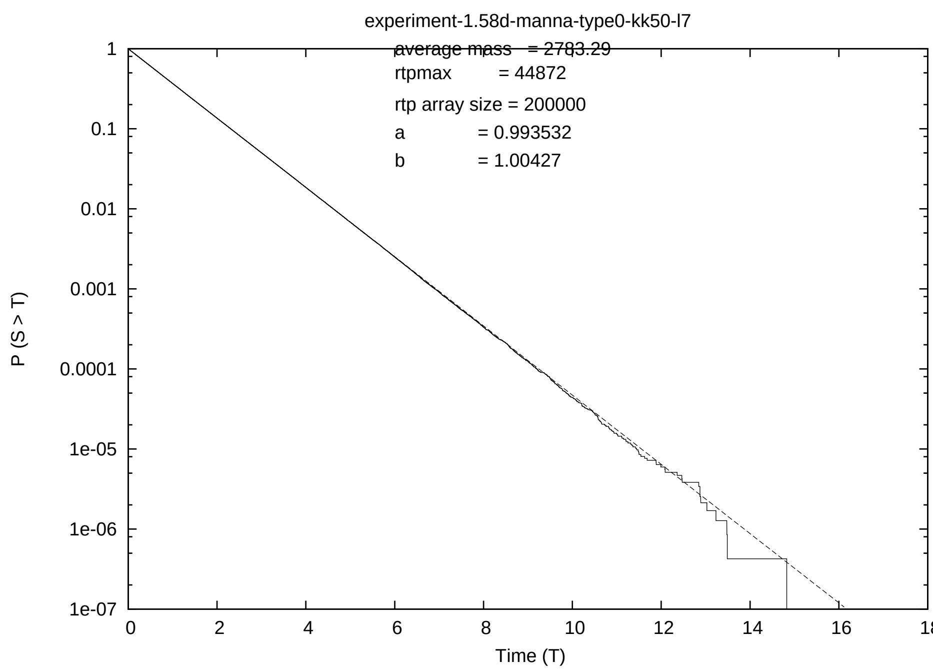

experiment-1.58d-manna-type0-kk50-l7
average mass = 2783.29
rtpmax = 44872
rtp array size = 200000
a = 0.993532
b = 1.00427
P (S > T)
Time (T)
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
2
4
6
8
10
12
14
16
18

Figure 6:

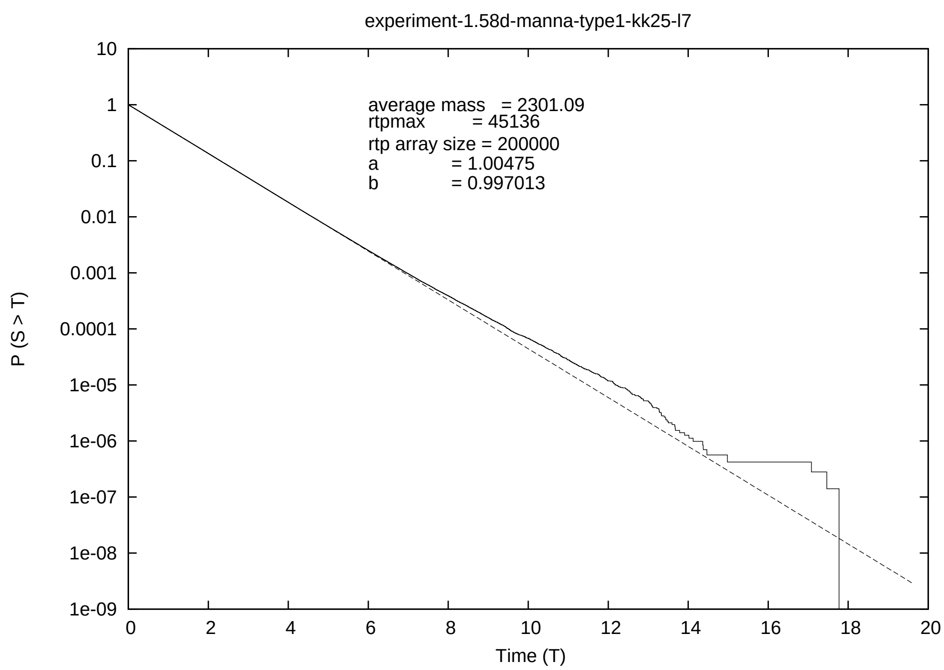
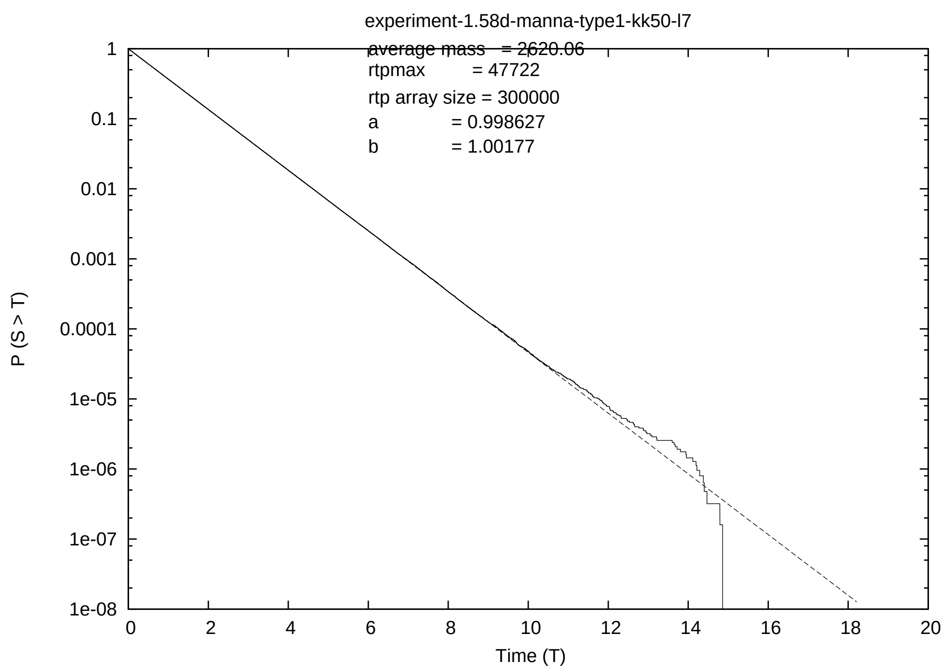
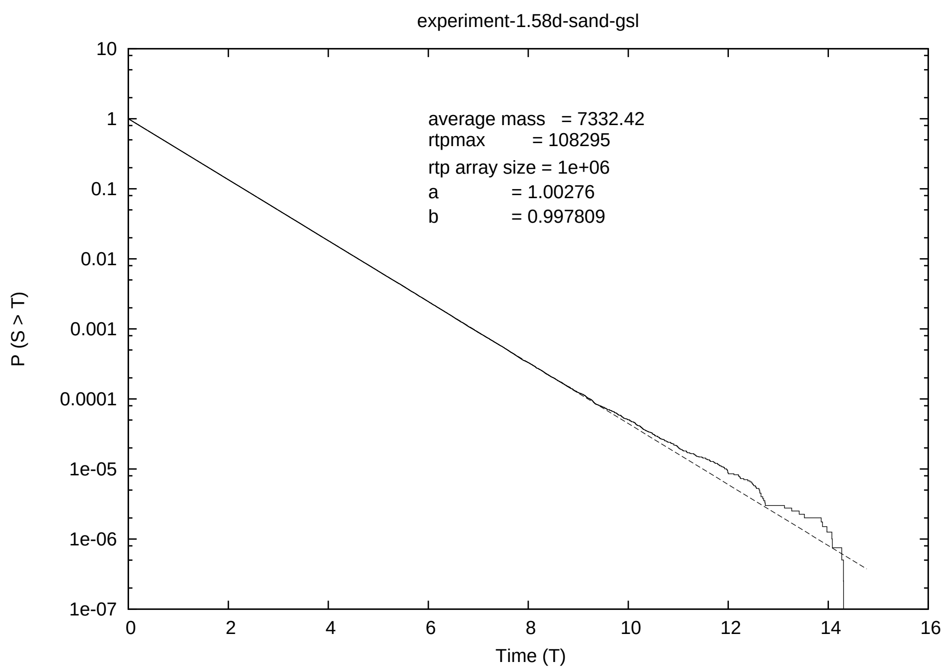

experiment-1.58d-manna-type1-kk25-l7
average mass = 2301.09
rtpmax = 45136
rtp array size = 200000
a = 1.00475
b = 0.997013
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
0
2
4
6
8
10
12
14
16
18
20

Figure 7:



experiment-1.58d-manna-type1-kk50-l7
average mass = 2620.06
rtpmax = 47722
rtp array size = 300000
a = 0.998627
b = 1.00177
P (S > T)
Time (T)
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
0
2
4
6
8
10
12
14
16
18
20

Figure 8:



experiment-1.58d-sand-gsl
average mass = 7332.42
rtpmax = 108295
rtp array size = 1e+06
a = 1.00276
b = 0.997809
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
2
4
6
8
10
12
14
16

Figure 9:

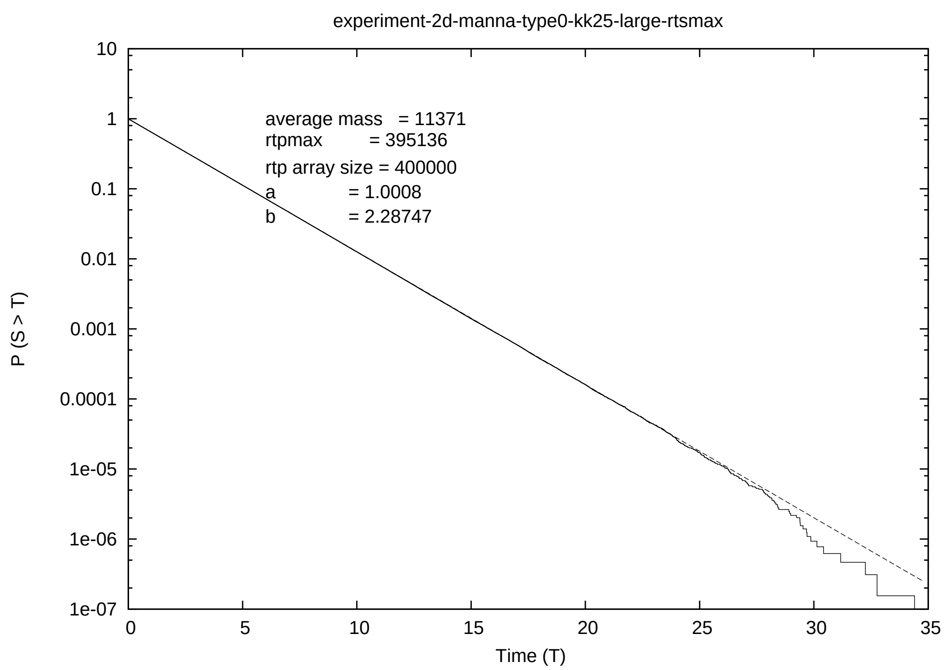

experiment-2d-manna-type0-kk25-large-rtsmax
average mass = 11371
rtpmax = 395136
rtp array size = 400000
a = 1.0008
b = 2.28747
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
5
10
15
20
25
30
35

Figure 10:

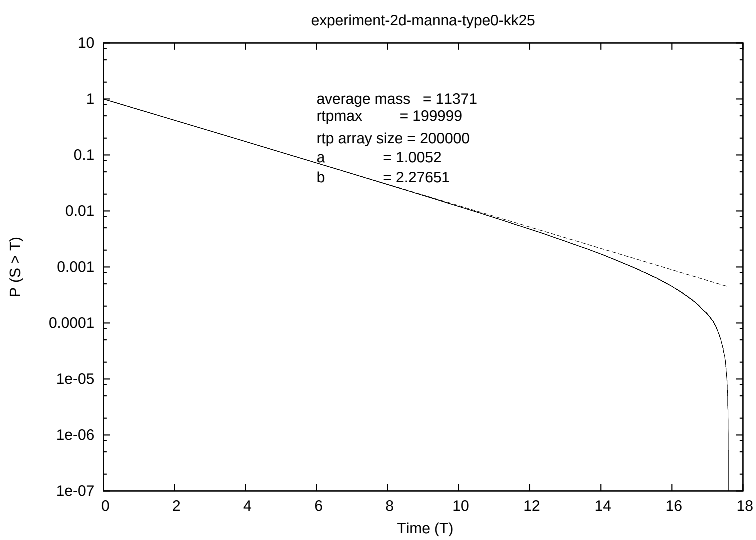

experiment-2d-manna-type0-kk25
average mass = 11371
rtpmax = 199999
rtp array size = 200000
a = 1.0052
b = 2.27651
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
2
4
6
8
10
12
14
16
18

Figure 11:

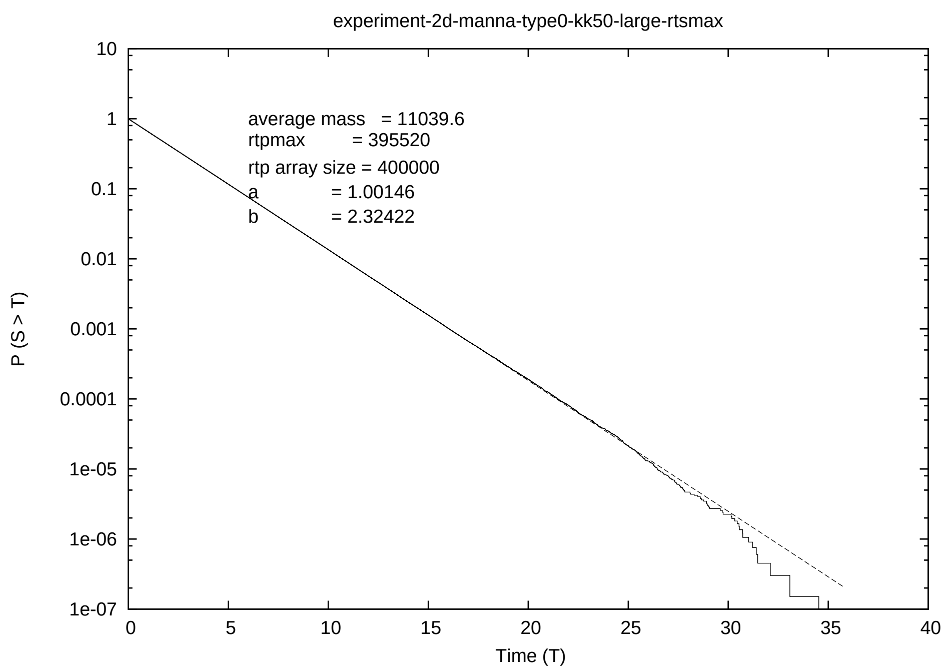

experiment-2d-manna-type0-kk50-large-rtsmax
average mass = 11039.6
rtpmax = 395520
rtp array size = 400000
a = 1.00146
b = 2.32422
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
5
10
15
20
25
30
35
40

Figure 12:

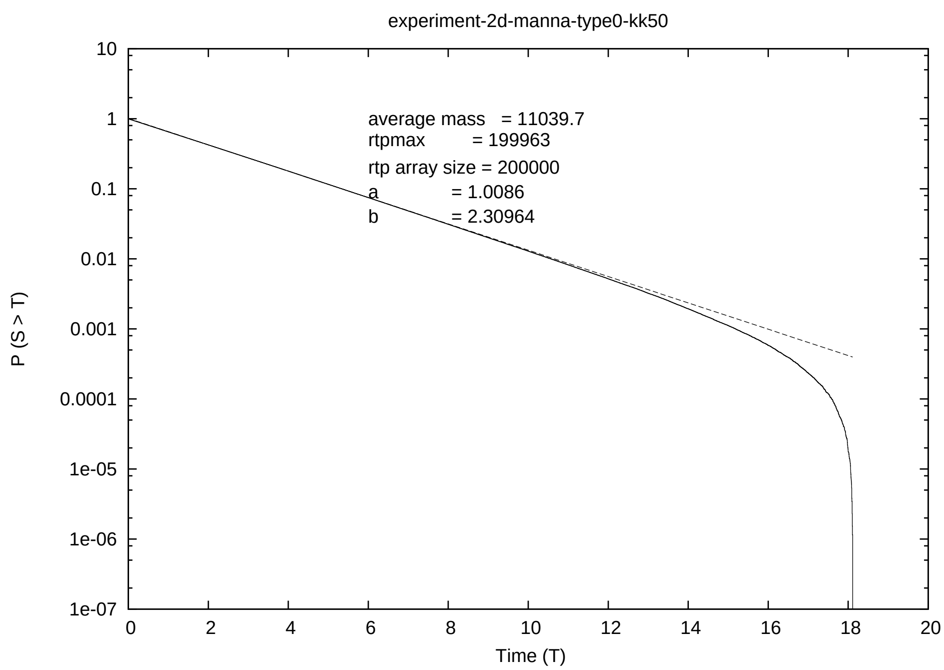
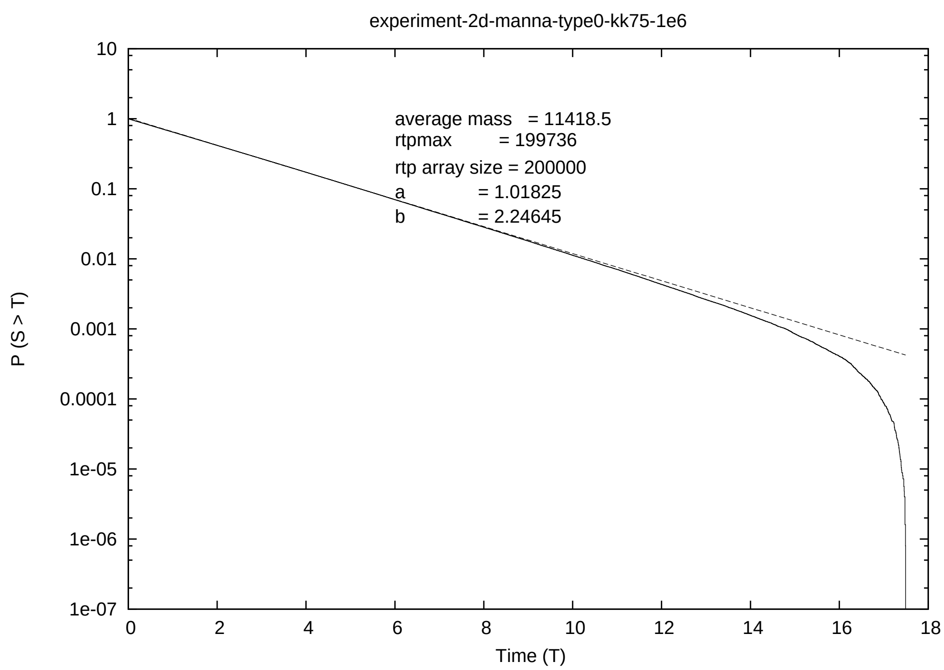

experiment-2d-manna-type0-kk50
average mass = 11039.7
rtpmax = 199963
rtp array size = 200000
a = 1.0086
b = 2.30964
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
2
4
6
8
10
12
14
16
18
20

Figure 13:



experiment-2d-manna-type0-kk75-1e6
average mass = 11418.5
rtpmax = 199736
rtp array size = 200000
a = 1.01825
b = 2.24645
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
2
4
6
8
10
12
14
16
18

Figure 14:

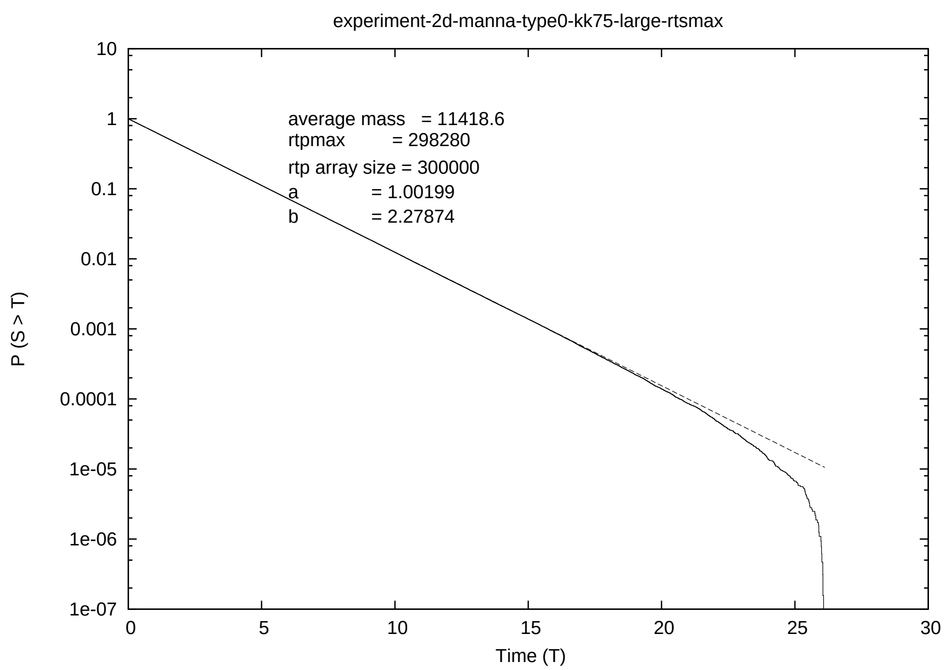

experiment-2d-manna-type0-kk75-large-rtsmax
average mass = 11418.6
rtpmax = 298280
rtp array size = 300000
a = 1.00199
b = 2.27874
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
5
10
15
20
25
30

Figure 15:

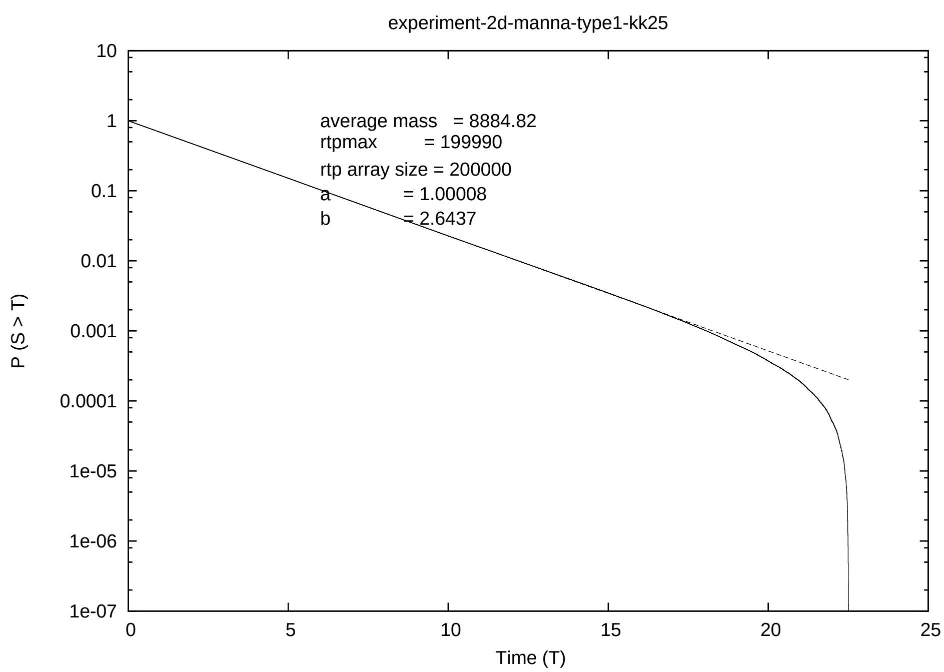
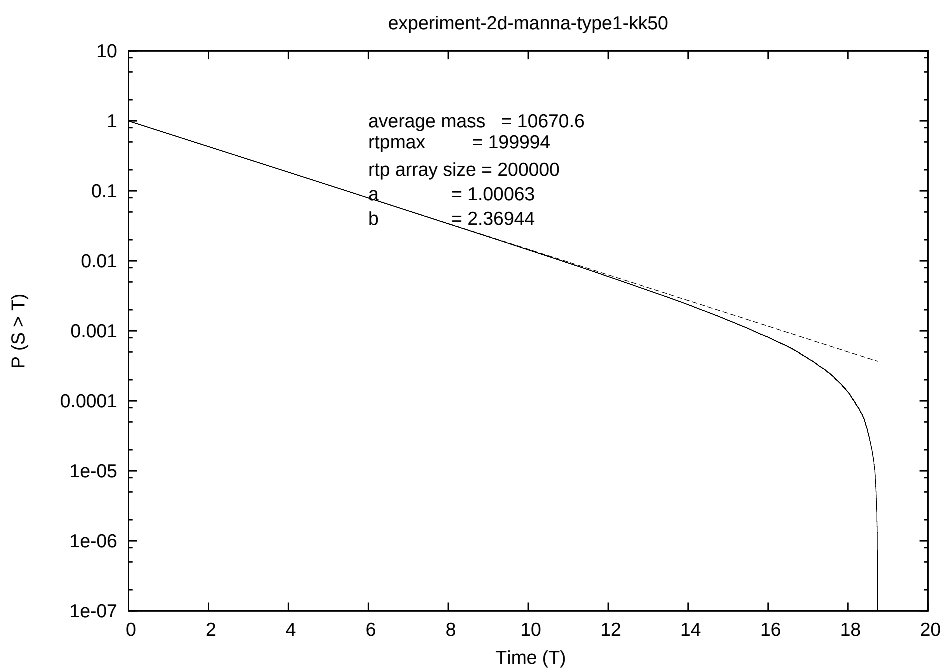

experiment-2d-manna-type1-kk25
average mass = 8884.82
rtpmax = 199990
rtp array size = 200000
a = 1.00008
b = 2.6437
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
5
10
15
20
25

Figure 16:



experiment-2d-manna-type1-kk50
average mass = 10670.6
rtpmax = 199994
rtp array size = 200000
a = 1.00063
b = 2.36944
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
2
4
6
8
10
12
14
16
18
20

Figure 17:

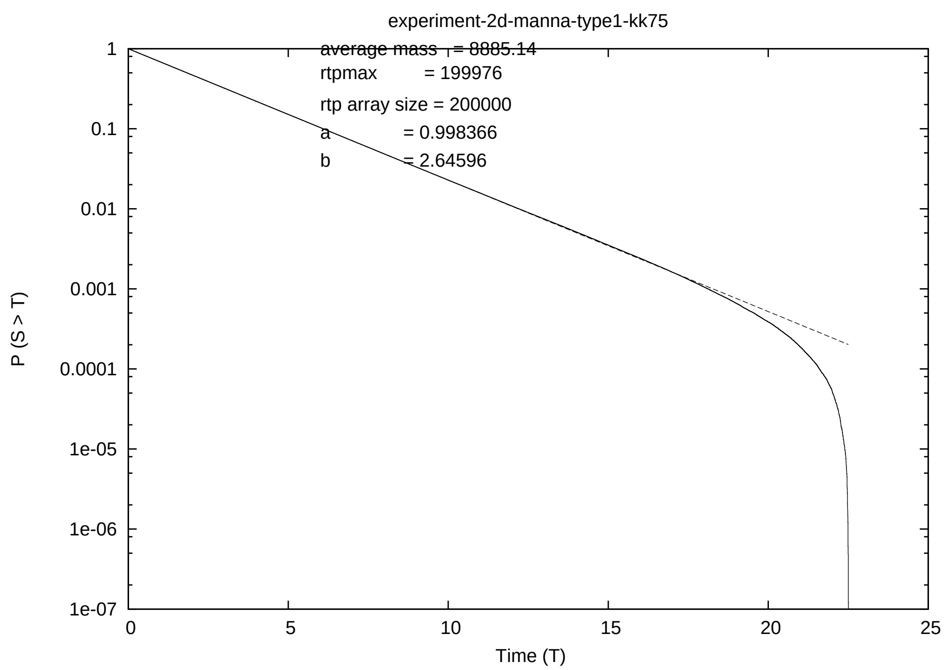
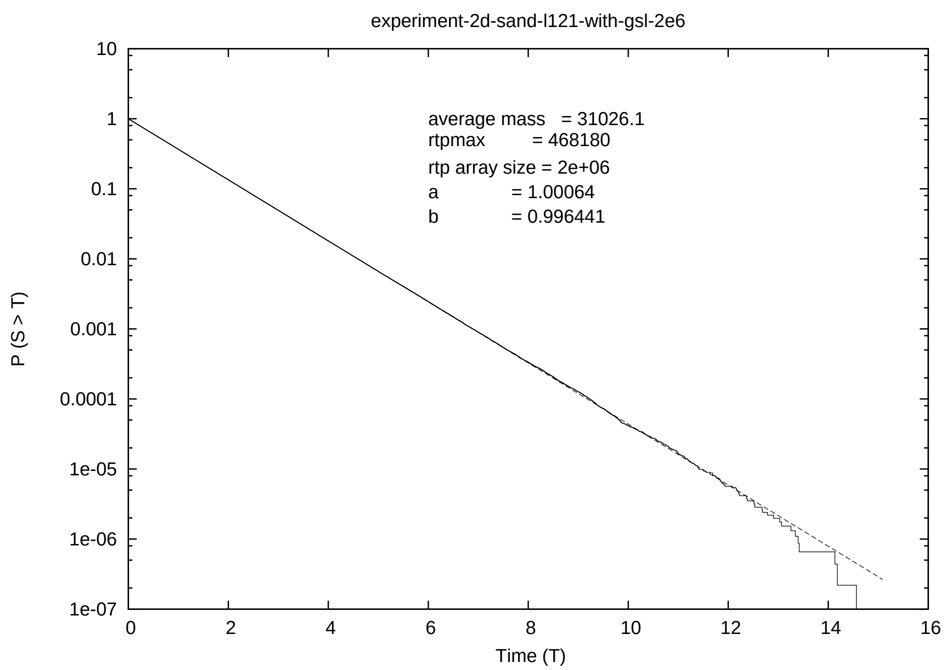

experiment-2d-manna-type1-kk75
average mass = 8885.14
rtpmax = 199976
rtp array size = 200000
a = 0.998366
b = 2.64596
P (S > T)
Time (T)
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
5
10
15
20
25

Figure 18:



experiment-2d-sand-l121-with-gsl-2e6
average mass = 31026.1
rtpmax = 468180
rtp array size = 2e+06
a = 1.00064
b = 0.996441
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
2
4
6
8
10
12
14
16

Figure 19:

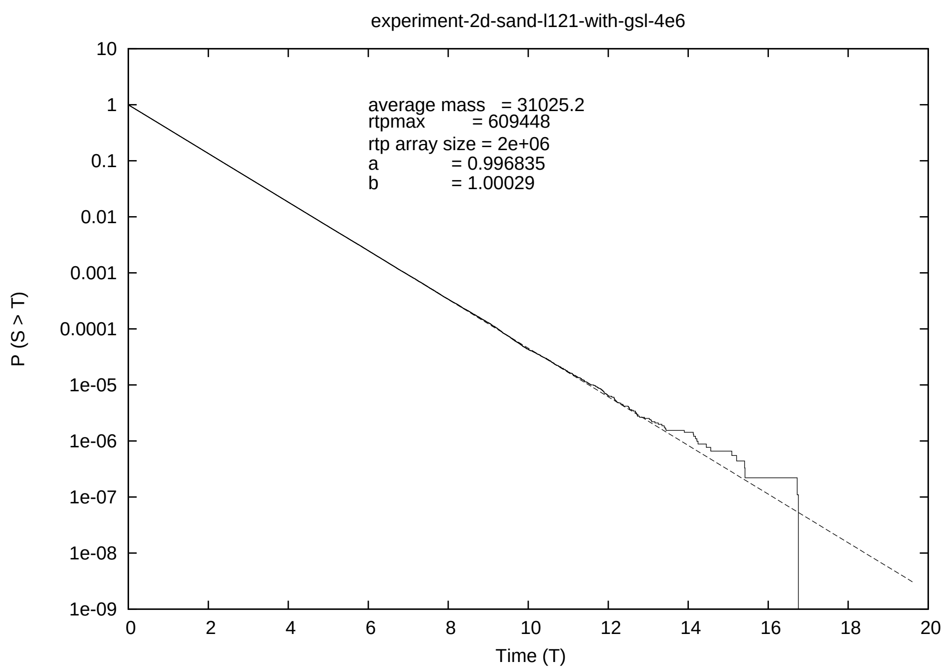

experiment-2d-sand-l121-with-gsl-4e6
average mass = 31025.2
rtpmax = 609448
rtp array size = 2e+06
a = 0.996835
b = 1.00029
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
0
2
4
6
8
10
12
14
16
18
20

Figure 20:

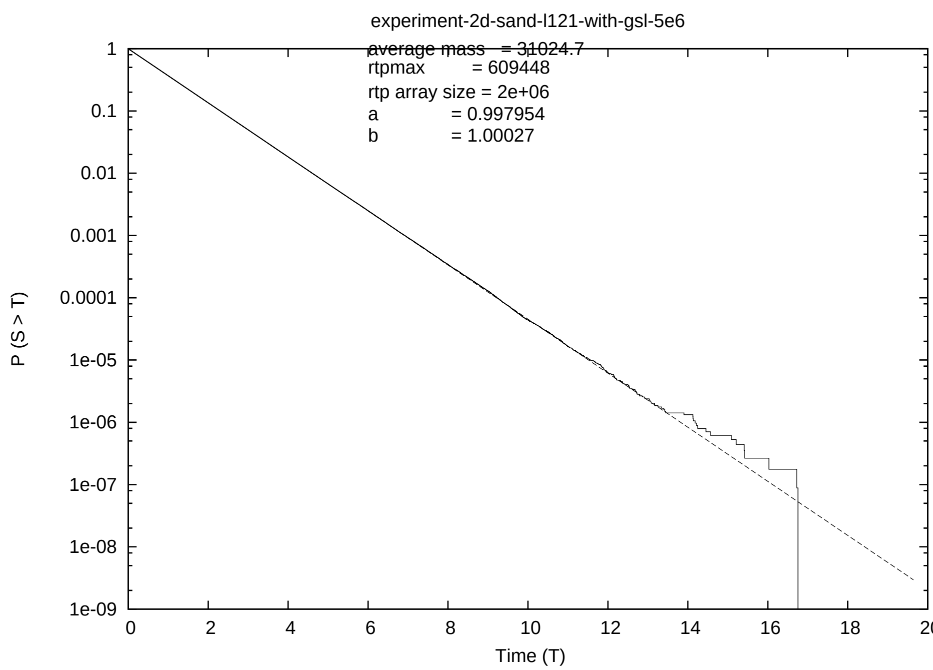

experiment-2d-sand-l121-with-gsl-5e6
average mass = 31024.7
rtpmax = 609448
rtp array size = 2e+06
a = 0.997954
b = 1.00027
P (S > T)
Time (T)
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
0
2
4
6
8
10
12
14
16
18
20

Figure 21:

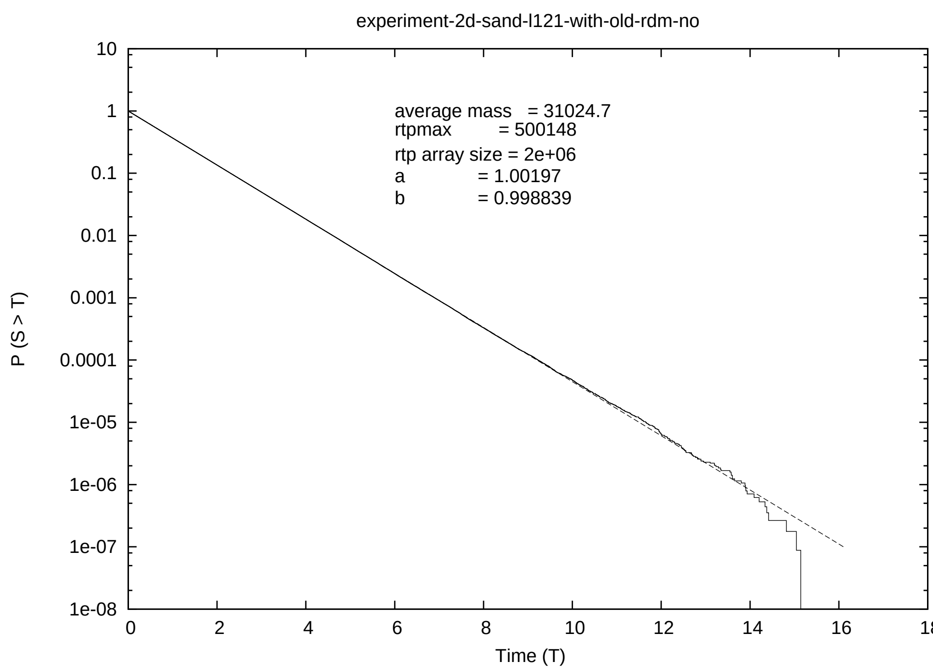

experiment-2d-sand-l121-with-old-rdm-no
average mass = 31024.7
rtpmax = 500148
rtp array size = 2e+06
a = 1.00197
b = 0.998839
P (S > T)
Time (T)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
0
2
4
6
8
10
12
14
16
18

Figure 22:

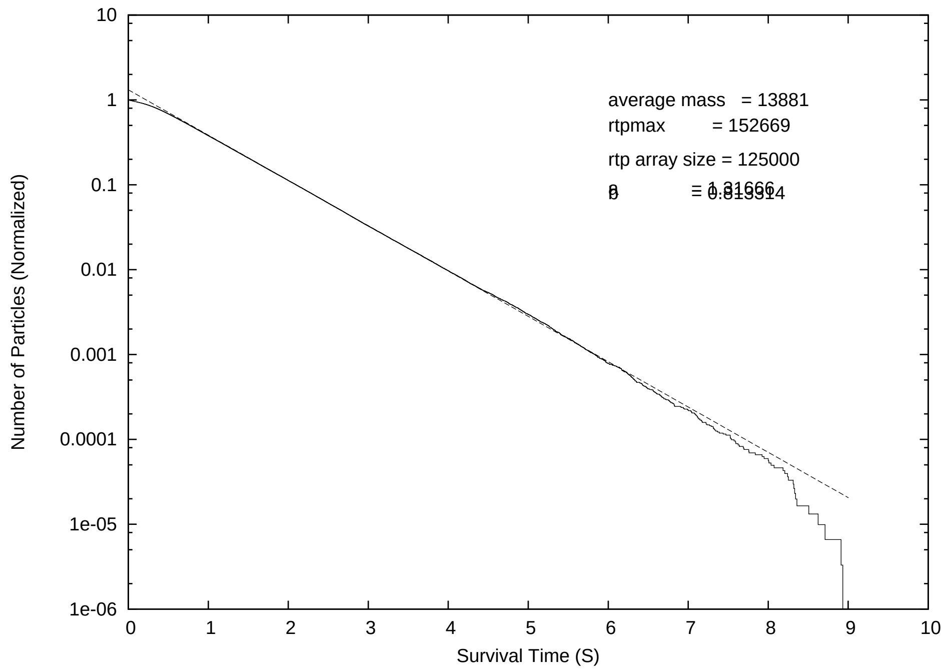

average mass = 13881
rtpmax = 152669
rtp array size = 125000
a = 1.31666
b = 0.815314
Number of Particles (Normalized)
Survival Time (S)
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
0
1
2
3
4
5
6
7
8
9
10

Figure 23:

4 Conclusions

The residence time follows a power law relation which is seen in systems which are 2–dimensional. This is unexpected from the intuitive expectation as the network for fractal systems is different.

We wre still tryng to establish some analytical solution which matches with the experiment. This may take some time.

After a suitable analytical model is obtained or the possibility of obtaining it is negated we plan to publish the results of our experiments.

5 Further Work

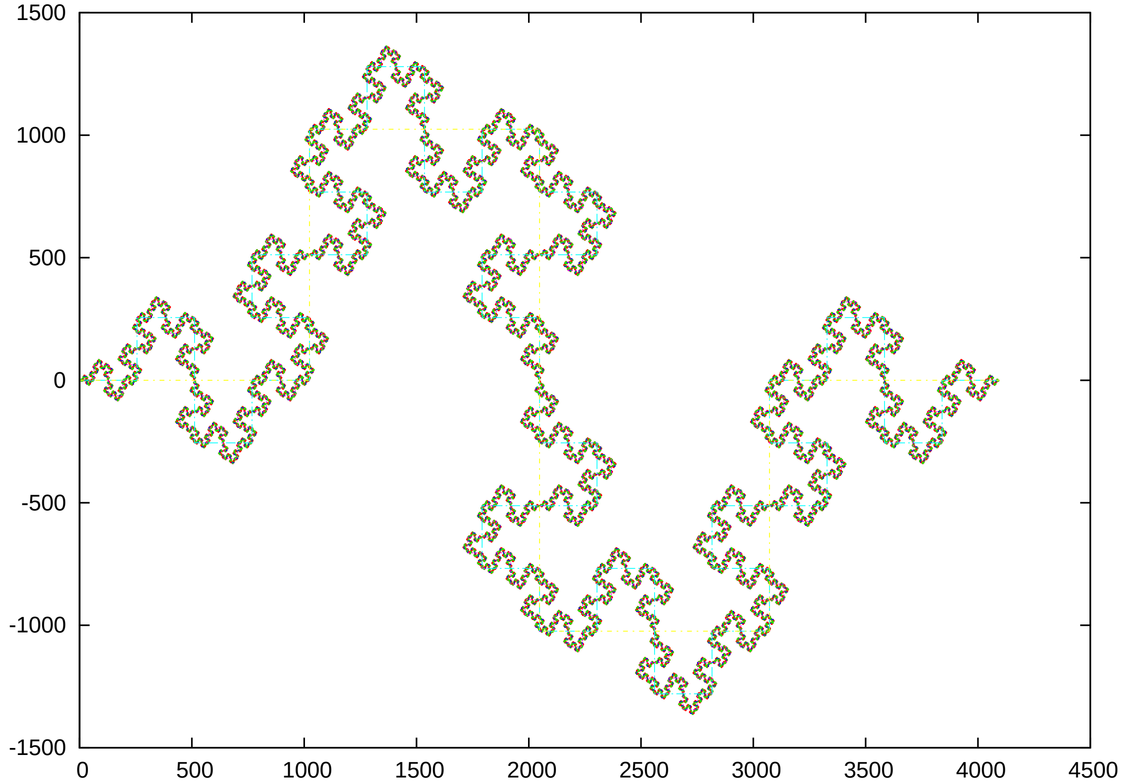


Figure 24: We represent the von Koch curve which is still being modelled.

We plan to continue experiments with other structures which have complex networks and also fractal or (irregular) boundaries to investigate whether the boundary has more of a role to play in the distribution of residence times. We have selected the von Koch curve for this purpose. The reason is that the internal structure is easily embedded in 2–dmensions, which means if the results are different we can investigate with more efforts the effect of it's boundary. .

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Devraj Pawar (Principal Investigator)