Hindi vidya Prachar Samiti's

Ramniranjan Jhunjhunwala College

of Arts, Science & Commerce

(Autonomous College)

Affiliated to

UNIVERSITY OF MUMBAI

Syllabus for the T.Y.B.Sc.

Program: B.Sc. Mathematics

Program Code: RJSUMAT

Choice based Credit System (CBCS)

With effect from the academic year 2018-19

Semester V

Paper 1: Multivariable Calculus and Analysis				
Course Code	Unit	Topics	Credits	L/week
RJSUMAT501	1	Line Integrals	2.5	3
	2	Surface Integrals		
	3	Sequence and Series of Functions		
Paper 2: Algebra-V				
RJSUMAT502	1	Quotient Spaces and Orthogonal Linear Transformations	2.5	3
	2	Eigenvalues and eigen vectors		
	3	Diagonalisation		
Paper 3: Topology of Metric Spaces-I				
RJSUMAT503	1	Metric spaces and open sets in metric spaces	2.5	3
	2	Closed sets and sequences in a metric space		
	3	Completeness and consequences of nested interval theorem		
Paper 4 : Number Theory and its Applications-I (Elective A)				
RJSUMAT504A	1	Congruences and Factorization	2.5	3
	2	Diophantine equations and their solutions		
	3	Primitive Roots and Cryptography		
Paper 4 : Numerical Analysis-I (Elective B)				
RJSUMAT504B	1	Error Analysis	2.5	3
	2	Transcendental and Polynomial equations		
	3	Linear Systems of Equations		
PRACTICALS				
RJSUMATP501	--	Practicals based on RJSUMAT501 and RJSUMAT502	3	6
RJSUMATP502	--	Practicals based on RJSUMAT503 and	3	6

		RJSUMAT504(A/B)		

Semester VI

Paper 1: Complex Analysis				
Course Code	Unit	Topics	Credits	L/week
RJSUMAT601	1	Analytic Functions	2.5	3
	2	Complex Integration		
	3	Complex Power Series		
Paper 2: Algebra-VI				
RJSUMAT602	1	Group Theory	2.5	3
	2	Ring Theory		
	3	Polynomial Rings and Field theory		
Paper 3: Topology of Metric Spaces-II				
RJSUMAT603	1	Compact sets	2.5	3
	2	Continuous functions on metric spaces		
	3	Connected Sets		
Paper 4 : Number Theory and its Applications-II (Elective A)				
RJSUMAT604A	1	Quadratic Reciprocity	2.5	3
	2	Continued Fractions		
	3	Arithmetic function and Special numbers		
Paper 4 : Numerical Analysis-II (Elective B)				
RJSUMAT604B	1	Interpolations	2.5	3
	2	Polynomial Approximations and Numerical Differentiation		
	3	Numerical Integration		
PRACTICALS				
RJSUMATP601	--	Practicals based on RJSUMAT601 and RJSUMAT602	3	6
RJSUMATP602	--	Practicals based on RJSUMAT603 and RJSUMAT604(A/B)	3	6

Note:

1. RJSUMAT501, RJSUMAT502, RJSUMAT503 are compulsory courses for Semester V.
2. Candidate has to opt one Elective Course from RJSUMAT504A/ RJSUMAT504B.
3. RJSUMAT601, RJSUMAT602, RJSUMAT603 are compulsory courses for Semester VI.
4. Candidate has to opt one Elective Course from RJSUMAT604A/ RJSUMAT604B.

Teaching Pattern for Semester V and VI:

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).

2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).

SEMESTER V

Paper 1: Multivariable Calculus and Analysis

Course code: RJSUMAT501

Unit 1: Line Integrals (15 Lectures)

Vector differential operators, gradient, curl, divergence, elementary identities involving gradient, curl and divergence. Paths (parametrized curves) in $\mathbb{R}^n$ (emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$), smooth and piecewise smooth paths, closed paths, equivalence and orientation preserving equivalence of paths. Definition of the line integral of scalar fields as well as vector fields over a piecewise smooth path, basic properties of line integrals including linearity, path-additivity and behavior under a change of parameters. Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals. Necessary and sufficient conditions for a vector field to be conservative, Green's theorem and its applications to evaluation of line integrals.

Unit 2: Surface Integrals (15 Lectures)

Parameteric representation of surfaces, Fundamental vector product-its definition and it being normal to the surface, area of parametric surfaces, definition of surface integrals of scalar fields as well as of vector fields defined on a surface. Stoke's theorem (proof assuming the general form of Green's theorem), examples. Gauss' divergence theorem (proof only in the case of cubical domains), examples.

Unit 3: Sequence and Series of Functions (15 Lectures)

Sequence of functions - pointwise and uniform convergence of sequences of real-valued functions. Series of functions, convergence of series of functions, Weierstrass M-test, examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous functions, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval, consequences of these properties for series of functions, term by term differentiation and integration.

Power series in $\mathbb{R}$ centered at origin and at some point in $\mathbb{R}$, radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions and basic properties of these functions.

Reference Books:

1. Tom M. Apostol, Calculus Vol. 2, second edition, John Wiley, India
2. Jerrold E. Marsden, Anthony J. Tromba, Alan Weinstein, Basic Multivariable Calculus, Indian edition, Springer-Verlag
3. Dennis G. Zill, Warren S. Wright, Calculus Early Transcendentals, fourth edition, Jones and Bartlett Publishers
4. R. R. Goldberg, Methods of Real Analysis, Indian Edition, Oxford and IBH publishing, New Delhi.
5. S.C. Malik, Savita Arora, Mathematical Analysis, third edition, New Age International Publishers, India.
6. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press.
7. Charles G. Denlinger, Elements of Real Analysis, student edition, Jones & Bartlett Publishers.
8. M. Thamban Nair, Calculus of One Variable, student edition, Ane Books Pvt. Ltd.
9. Russell A. Gordon, Real Analysis A First Course, Second edition, Addison-Wesley.

T.Y.B.Sc.	Semester V
RJSUMAT501 Paper I Multivariable Calculus and Analysis	Course Outcome 5.1 : 1. To define line integrals of scalar and vector fields, basic properties and conservative of vector field 2. To learn Fundamental Theorems of Calculus for line integrals, Green's theorem and their applications 3. To understand the concept of surface integrals for scalar and vector fields and some identities involving gradient, curl and divergence 4. To study two important theorems viz. Stoke's thereom and Gauss divergence theorem and examples 5. To study pointwise and uniform convergence of sequence and series of functions 6. To learn the conseuqence of uniform convergence on limit functions 7. To introduce the concept of power series and representation of elementary functions Learning Outcome : 1. Study of Line integral of scalar and vector fields 2. Surface area, surface integrals of scalar and vector fields, understanding relations between line integral, surface

	integral and double and triple integrals 3. Learning of sequence and series of functions and their consequences on continuity, differentiability and integrability

Paper 2: Algebra-V

Course code: RJSUMAT502

Unit 1: Quotient Spaces and Orthogonal Linear Transformations (15 Lectures)

Review of vector spaces over $\mathbb{R}$, sub spaces and linear transformation.

Quotient Spaces: For a real vector space V and a subspace W , the cosets $v + W$ and the quotient space $V/_W$, First Isomorphism theorem of real vector spaces (fundamental theorem of homomorphism of vector spaces), Dimension and basis of the quotient space $V/_W$, when V is finite dimensional.

Orthogonal transformations: Isometries of a real finite dimensional inner product space, Translations and Reflections with respect to a hyperplane, Orthogonal matrices over $\mathbb{R}$, Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space, Orthogonal transformation of $\mathbb{R}^2$, Any orthogonal transformation in $\mathbb{R}^2$ is a reflection or a rotation, Characterization of isometries as composites of orthogonal transformations and translation. Characteristic polynomial of an $n \times n$ real matrix. Cayley Hamilton Theorem and its Applications (Proof assuming the result A(adjA) = I_n for an $n \times n$ matrix over the polynomial ring $\mathbb{R}[t]$).

Unit 2: Eigenvalues and eigen vectors (15 Lectures)

Eigen values and eigen vectors of a linear transformation T: $V \to V$, where V is a finite dimensional real vector space and examples, Eigen values and Eigen vectors of $n \times n$ real matrices, The linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation.
The characteristic polynomial of an nxn real matrix and a linear transformation of a finite dimensional real vector space to itself, characteristic roots, Similar matrices, Relation with change of basis, Invariance of the characteristic polynomial and (hence of the) eigenvalues of similar matrices, Every square matrix is similar to an upper triangular matrix. Minimal Polynomial of a matrix, Examples like minimal polynomial of scalar matrix, diagonal matrix, similar matrix, Invariant subspaces.

Unit 3: Diagonalisation (15Lectures)

Geometric multiplicity and Algebraic multiplicity of eigen values of an $n \times n$ real matrix, An $n \times n$ matrix A is diagonalizable if and only if it has a basis of eigenvectors of A if and only if the sum of dimension of eigen spaces of A is n if and only if the algebraic and geometric multiplicities of eigen values of A coincide, Examples of non diagonalizable matrices, Diagonalisation of a linear transformation $T: V \to V$, where V is a finite dimensional real vector space and examples. Orthogonal diagonalisation and Quadratic Forms. Diagonalisation of real Symmetric matrices, Examples, Applications to real Quadratic forms, Rank and Signature of a Real Quadratic form, Classification of conics in $\mathbb{R}^2$ and quadric surfaces in $\mathbb{R}^3$. Positive definite and semi definite matrices, Characterization of positive definite matrices in terms of principal minors.

Recommended Books.

1. S. Kumaresan, Linear Algebra, A Geometric Approach.
2. Ramachandra Rao and P. Bhimasankaram, Tata McGraw Hill Publishing Company.

Additional Reference Books

1. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.
2. L. Smith, Linear Algebra, Springer.
3. M. R. Adhikari and Avishek Adhikari, Introduction to linear Algebra, Asian Books Private Ltd.
4. K Hoffman and Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
5. Inder K Rana, Introduction to Linear Algebra, Ane Books Pvt. Ltd.

T.Y.B.Sc.	Semester V
RJSUMAT502 Paper II Algebra-V	Course Outcome 5.2 : 1. Quotient spaces and Isomorphism of real vector spaces. 2. Orthogonal transformations ,Isometries, Characteristic polynomials and Cayley Theorm. 3. Eigen values and eigen vectors, similar matrices . 4. Diagonalisation of a matrix with respect to eigen values/eigen vectors. 5. Quadratic forms. Learning Outcome : 1. Application of First, second and Third Isomorphism theorem , cayley theorem. 2. Deatailed study of similar matrices and its relation with change of basis. 3. Applications of Quadratic form.

Paper 3: Topology of Metric Spaces-I
Course Code: RJSUMAT503

Unit 1: Metric spaces, open balls and open sets in metric spaces (15 Lectures)
Definition, examples of metric spaces $\mathbb{R}$; $\mathbb{R}^2$, Euclidean space $\mathbb{R}^n$ with its Euclidean, sup and sum metric, $\mathbb{C}$(complex numbers), the spaces ℓ^1, ℓ^2 and ℓ^∞ of sequences and the space C[a, b] of real valued continuous functions on [a, b]. Discrete metric space. Finite metric spaces.
Normed linear spaces, distance metric induced by the norm.
Metric subspaces, Product of two metric spaces. Open balls, closed balls, closed sets in a metric space, examples and properties. Hausdorff property. Interior of a set. Structure of an open set in $\mathbb{R}$, Equivalent metrics.

Unit 2: Limit points of a set and sequences in a metric space (15 Lectures)
Limit point of a set, a closed set contains all its limit points, Closure of a set and boundary of a set. Distance of a point from a set, distance between two sets, diameter of a set in a metric space and bounded sets in a metric space.
Definition and examples - Sequences, Convergent sequence, Cauchy sequence, and subsequences in metric space and properties.
Characterization of limit points and closure points in terms of sequences. Dense subsets in a metric space and Separability.

Unit 3: Completeness and consequences of nested interval theorem (15 Lectures)
Definition of complete metric spaces, examples of complete metric spaces, completeness property in subspaces, Cantor's Intersection Theorem, Nested Interval theorem in $\mathbb{R}$, Applications of Nested interval Theorem:
(i) The set of real Numbers is uncountable.
(ii) Density of rational Numbers (between any two real numbers there exists a rational number)
(iii) Intermediate Value theorem for $\mathbb{R}$
(iv) Bolzano-Weierstrass theorem for $\mathbb{R}$
(v) Heine-Borel Theorem for $\mathbb{R}$

Reference books:
1. S. Kumaresan, Topology of Metric spaces.
2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi.
3. Expository articles of MTTS programme.

Other References :
1. W. Rudin, Principles of Mathematical Analysis.

2. T. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi.
3. E. T. Copson, Metric Spaces, Universal Book Stall, New Delhi.
4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi
5. P.K.Jain. K. Ahmed, Metric Spaces, Narosa, New Delhi.
6. W. Rudin, Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland.
7. D. Somasundaram, B. Choudhary, A first Course in Mathematical Analysis, Narosa, New Delhi
8. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, New York.
9. W. A. Sutherland, Introduction to metric & topological spaces, Second Edition, Oxford.

T.Y.B.Sc.	Semester V Theory
RJSUMAT503 Paper III Topology of Metric Spaces-I	Course Outcome 5.3: 1. Metric spaces, normed spaces and examples 2. To learn open sets, closed sets, limit points of a set, closure of a set 3. Sequences in metric spaces and their properties 4. To study complete metric space and Nested Interval Theorem and its applications 5. Separability and dense subsets of metric spaces Learning Outcome : 1. To understand Metric spaces, normed spaces 2. Convergent and Cauchy sequences in metric spaces 3. Completeness of metric space and consequences of Nested Interval Theorem

Paper 4: Number Theory and its applications – I [Elective A]
Course Code: RJSUMAT504A

Unit 1: Congruences and Factorization (15 Lectures)

Review of Divisibility, Primes and the fundamental theorem of Arithmetic.

Congruences, Complete residue system modulo m, Reduced residue system modulo m, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem, Linear congruences, Simultaneous linear congruences in two variables. The Chinese remainder Theorem, Congruences of Higher degree, The Fermat-Kraitchik Factorization Method.

Unit 2: Diophantine equations and their solutions (15 Lectures)

The linear Diophantine equation ax + by = c. The equation $x^2+ y^2= z^2$, Primitive Pythagorean triple and its characterisation, The equations $x^4+ y^4= z^2$ and $x^4+y^4= z^4$ have no solutions (x; y; z) with $xyz \neq 0$. Fermat's two squares theorem, sum of three squares, Lagrange's four squares theorem.

Unit 3: Primitive Roots and Cryptography (15 Lectures)

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as shift cipher, Affine cipher, Hill's cipher, Vigenere cipher, digraph transformations. Concept of Public Key Cryptosystem; RSA Algorithm, Digital Signatures, ElGamal Cryptosystem.

Recommended Books:

1. Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc.
2. David M. Burton, An Introduction to the Theory of Numbers, Tata McGraw Hill Edition.
3. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press.
4. Neville Robins, Beginning Number Theory, Narosa Publications.
5. S.D. Adhikari, An introduction to Commutative Algebra and Number Theory, Narosa Publishing House.
6. N. Koblitz, A course in Number theory and Cryptography, Springer.
7. M. Artin, Algebra, Prentice Hall.
8. K. Ireland, M. Rosen, A classical introduction to Modern Number Theory, Second edition, Springer Verlag.
9. William Stallings, Cryptology and network security, Pearson Education.

10. T. Koshy, Elementary number theory with applications, 2nd edition, Academic Press.
11. A. Baker, A comprehensive course in number theory, Cambridge.

T.Y.B.Sc.	Semester V Theory
RJSUMAT504A Paper IV Number Theory and its applications – I	Course Outcome 5.4A: 1. To understand applications of Fermat's theorem, Euler's theorem, Wilson's theorem 2. To understand applications of Chinese remainder theorem and methods of factorization 3. To learn use of primes and congruences in the field of Cryptography 4. To understand concepts of primitive roots. 5. To learn solvability of Pythagorean triples and linear non linear Diophantine equations 6. To learn concept of expressing a natural number as a sum of two squares, three squares and four squares. Learning Outcome : 1. To be able to implement elementary methods of cryptography 2. Should be able to handle higher power in integers 3. Understanding of classical problems in number theory

Paper 4: Numerical Analysis – I [Elective B]
Course Code: RJSUMAT504B

Unit I. Errors Analysis and Transcendental & Polynomial Equations (15L)

Measures of Errors: Relative, absolute and percentage errors. Types of errors: Inherent error, Round-off error and Truncation error. Taylors series example. Significant digits and numerical stability. Concept of simple and multiple roots. Iterative methods, error tolerance, use of intermediate value theorem. Iteration methods based on first degree equation: Newton-Raphson method, Secant method, Regula-Falsi method, Iteration Method. Condition of convergence and Rate of convergence of all above methods.

Unit II. Transcendental and Polynomial Equations (15L)

Iteration methods based on second degree equation: Muller method, Chebyshev method, Multipoint iteration method. Iterative methods for polynomial equations; Descarts rule of signs, Birge-Vieta method, Bairstrow method. Methods for multiple roots. Newton-Raphson method.

System of non-linear equations by Newton- Raphson method. Methods for complex roots. Condition of convergence and Rate of convergence of all above methods.

Unit III. Linear System of Equations (15L)

Matrix representation of linear system of equations. Direct methods: Gauss elimination method.

Pivot element, Partial and complete pivoting, Forward and backward substitution method, Triangularization methods-Doolittle and Crouts method, Choleskys method. Error analysis of direct methods. Iteration methods: Jacobi iteration method, Gauss-Siedal method. Convergence analysis of iterative method. Eigen value problem, Jacobis method for symmetric matrices Power method to determine largest eigenvalue and eigenvector.

Recommended Books

1. Kendall E. and Atkinson, An Introduction to Numerical Analysis, Wiley.

2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.

3. S.D. Conte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGraw Hill International Book Company.

4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.

5. Hildebrand F.B., Introduction to Numerical Analysis, Dover Publication, NY.

6. Scarborough James B., Numerical Mathematical Analysis, Oxford University Press, New Delhi.

T.Y.B.Sc.	Semester V Theory
RJSUMAT504B Paper IV Numerical Analysis – I	Course Outcome 5.4B: 1. To understand Newton-Raphson method, Secant method, Regula-Falsi method, and their rate of convergence. 2. To learn Iteration methods: Muller method, Chebyshev method, Multipoint iteration method and their rate of convergence 3. To learn Doolittle and Crouts method, Choleskys method, Jacobi iteration method, Gauss-Siedal method and convergence analysis Learning Outcome : 1. To understand various types of errors and their sources 2. To learn methods of finding approximate roots of an equation 3. To learn methods of finding solutions of simultaneous linear equations

Course: Practicals based on RJSUMAT501 and RJSUMAT502
Course code: RJSUMATP501

List of suggested practicals based on RJSUMAT501:

1. Line integrals of scalar and vector fields
2. Green's theorem, conservative field and applications
3. Evaluation of surface integrals
4. Stokes and Gauss divergence theorem
5. Pointwise and uniform convergence of sequence of functions
6. Pointwise and uniform convergence of series of functions
7. Miscellaneous theoretical questions based on full paper

Suggested Practicals based on RJSUMAT502

1. Quotient Spaces, Orthogonal Transformations.
2. Cayley Hamilton Theorem and Applications
3. Eigen Values & Eigen Vectors of a linear Transformation/ Square Matrices
4. Similar Matrices, Minimal Polynomial, Invariant Subspaces
5. Diagonalisation of a matrix
6. Orthogonal Diagonalisation and Quadratic Forms.
7. Miscellaneous Theory Questions

T.Y.B.Sc.	Semester VI Practical
RJSUMATP501 Based on RJSUMAT501 and RJSUMAT502	Course Outcome: 1. To compute line integrals of scalar and vector fields 2. To evaluate given integral using Green's theorem 3. To evaluate surface integrals 4. To examine pointwise and uniform convergence 5. Quotient Spaces, Orthogonal Transformations, Eigen

	values/vectors 6. Similar Matrices, Minimal Polynomial, Diagonalisation of a matrix and Quadratic Forms Learning Outcome : 1. To know elementary identities involving various operators 2. Applications of Green's theorem 3. Applications of Stoke's and divergence theorem 4. To learn to classify pointwise and uniform convergence Learn to solve varieties of problems based on Quotient spaces, similar matrices, orthogonal diagonalisation of a matrix and quadratic forms etc. and able to co-relate among them.

Course: Practicals based on RJSUMAT503 and RJSUMAT504A/ RJSUMAT504B
Course code: RJSUMATP502

List of suggested practicals based on RJSUMAT503:

1. Example of metric spaces, normed linear spaces

2. Sketching of open balls in $\mathbb{R}^2$ and open sets in metric spaces/ normed linear spaces, interior of a set, subspaces

3. Closed sets, sequences in a metric space

4. Limit points, dense sets, separability, closure of a set, distance between two sets.

5. Complete metric space

6. Cantor's Intersection theorem and its applications

7. Miscellaneous theory questions from all unit

List of suggested practicals based on RJSUMAT504A:

1. Fermat's theorem, Wilson's theorem, Euler's theorem

2. Chinese remainder theorem, linear and higher order congruences, factorization

3. Linear Diophantine equations

4. Pythagorean triples, sum of two squares, three squares, four squares

5. Primitive roots, shift cipher, affine cipher, Hill cipher

6. Vigenere Cipher, Digraph transformations, Public key cryptosystems

7. Miscellaneous theoretical questions from all units

List of suggested practicals based on RJSUMAT504B:

1. Newton-Raphson method, Secant method, Regula-Falsi method, Iteration Method

2. Muller method, Chebyshev method, Multipoint iteration method

3. Descarts rule of signs, Birge-Vieta method, Bairstrow method

4. Gauss elimination method, Forward and backward substitution method,

5. Triangularization methods-Doolittles and Crouts method, Choleskys method

6. Jacobi iteration method, Gauss-Siedal method

7. Eigen value problem: Jacobis method for symmetric matrices and Power method to determine largest eigenvalue and eigenvector

T.Y.B.Sc.	Semester VI Practical
RJSUMATP502 Based on RJSUMAT503 and RJSUMAT504 (A/B)	Course Outcome: 1. To learn concept of metric space and normed spaces through examples 2. To understand sequence in metric spaces and its properties 3. To study various type of points in a metric space 4. To implement the Chinese Remainder Theorem, Fermat's theorem, Euler's theorem

	5. To implement affine cipher, RSA cryptosystem, ElGamal cryptosystem 6. To use Newton Raphson method, secant method, multipoint iteration method Learning Outcome : 1. Completeness of metric spaces 2. Geometric structure on vector spaces 3. To be able to solve simultaneous linear congruences and polynomial congruences 4. Finding primitive roots and its use in constructing Elgamal Cryptosystem 5. Finding approximate roots of a polynomial and transcendental equations 6. Finding approximate solutions of simultaneous linear equations

SEMESTER VI

Paper 1: Complex Analysis

Course code: RJSUMAT601

Unit 1: Analytic Functions (15 Lectures)

Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivre's formula, $\mathbb{C}$ as a metric space, bounded and unbounded sets, point at infinity-extended complex plane, sketching of a set in complex plane (No questions to be asked).

Limit at a point, theorems on limits, convergence of sequences of complex numbers and results using properties of real sequences, functions from $\mathbb{C}$ to $\mathbb{C}$, real and imaginary part of functions, continuity at a point and algebra of continuous functions, derivative of $f:\mathbb{C}\to\mathbb{C}$; comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic function, and algebra of analytic functions, chain rule, harmonic functions and harmonic conjugate.

Unit 2: Complex Integration (15 Lectures)

Exponential function and its properties, trigonometric functions, hyperbolic functions, Mobius transformations.

Evaluating the line integral $\int f(z)dz$ over $|z-z_0|=r$, The Cauchy integral formula, Cauchy integral theorem, consequences of the Cauchy integral formula, derivative of analytic functions, Liouville's theorem, Application to the Fundamental Theorem of Algebra, Maximum modulus theorem.

Unit 3: Complex Power Series (15 Lectures)

Taylor's theorem for analytic functions, power series of complex numbers and related results, radius of convergence, disc of convergence, uniqueness of series representation, Laurent series, definition of isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, Cauchy residue theorem and calculation of residue, applications of residues.

Reference books:

1. James Ward Brown, Ruel V. Churchill, Complex variables and applications, seventh edition, McGraw Hill
2. Alan Jeffrey, Complex Analysis and Applications, second edition, CRC Press
3. Reinhold Remmert, Theory of Complex Functions, Springer

4. S. Ponnusamy, Foundations of Complex Analysis, second edition, Narosa Publishing House
5. Richard A. Silverman, Introductory Complex Analysis, Prentice-Hall, Inc.
6. Dennis G. Zill, Patrick D. Shanahan, Complex Analysis A First Course with Applications, third edition, Jones & Bartlett
7. H.S. Kasana, Complex Variables Theory and Applications, second edition, PHI Learning Private Ltd.
8. Jerrold E. Marsden, Michael Hoffman, Basic Complex Analysis, third edition, W.H. Freeman, New York

T.Y.B.Sc.	Semester V
RJSUMAT601 Paper I Complex Analysis	Course Outcome 6.1: 1. To study Limit and continuity, differentiability and analyticity of complex functions 2. To study elementary functions in complex plane and transfomations 3. To evaluate complex integration using Cauchy integral formula 4. To learn power series of complex numbers including Taylor's series and Laurent's series and different types of singularities 5. To compute residues and its applications Learning Outcome : 1. To be able to identify complex differentiability, analyticity of complex functions using definition and C-R equations 2. Cauchy theory of complex integration and its applications 3. To understand complex power series and types of singularities

PAPER II - ALGEBRA-VI
Course code : RJSUMAT602

Unit 1: Group Theory (15Lectures)
Review of Groups , Subgroups, Abelian groups, Order of a group, Finite and infinite groups, Cyclic groups, The Center $Z(G)$ of a group G. Cosets, Lagranges theorem, Group homomorphisms, isomorphisms, automorphisms, inner automorphisms (No questions to be asked).

Normal subgroups: Normal subgroups of a group, definition and examples including center of a group, Quotient group, Alternating group A_n, Cycles. Listing normal subgroups of A_4, S_3. First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups), Second Isomorphism theorem, third Isomorphism theorem, Cayley's theorem, External direct product of a group, Properties of external direct products, Order of an element in a direct product, criterion for direct product to be cyclic, Classification of groups of order ≤ 7.

Unit 2: Ring Theory (15Lectures)
Motivation: Integers & Polynomials.
Definitions of a ring (The definition should include the existence of a unity element), zero divisor, unit, the multiplicative group of units of a ring. Basic Properties & examples of rings, including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, M_n(\mathbb{R}), \mathbb{Q}[X], \mathbb{R}[X], \mathbb{C}[X]$, $\mathbb{Z}[i], \mathbb{Z}[\sqrt{-5}\,], \mathbb{Z}_n$
Definitions of Commutative ring, integral domain (ID), Division ring, examples. Theorem such as: A commutative ring R is an integral domain if and only if for $a, b, c \in R \; with \; a \neq 0$ the relation $ab = ac$ implies that $b = c$. Definitions of Subring, examples. Ring homomorphisms, Properties of ring homomorphisms, Kernel of a ring homomorphism, Ideals, Operations on ideals and Quotient rings, examples. Factor theorem, First and second Isomorphism theorems for rings, Correspondence Theorem for rings: If $f : R \to R'$ is a surjective ring homomorphism, then there is a 1-1 correspondence between the ideals of R containing the ker(f) and the ideals of R'. Definitions of characteristic of a ring, Characteristic of an ID.

Unit 3: Polynomial Rings and Field theory (15 Lectures)
Principal ideal, maximal ideal, prime ideal, characterization of the prime and maximal ideals in terms of quotient rings. Polynomial rings, R[X] when R is an integral domain/field. Divisibility in an Integral Domain, Definitions of associates, irreducible and primes. Prime (irreducible) elements in $\mathbb{R}[X], \mathbb{Q}[X]; \; \mathbb{Z}_p[X]$. Eisenstein's criterion for irreducibility of a polynomial over

$\mathbb{Z}$. Prime and maximal ideals in polynomial rings. Definition of field, subfield and examples, characteristic of fields. Any field is an ID and a finite ID is a field. Characterization of fields in terms of maximal ideals, irreducible polynomials. Construction of quotient field of an integral domain (Emphasis on $\mathbb{Z}, \mathbb{Q}$). A field contains a subfield isomorphic to $\mathbb{Z}_p$ or $\mathbb{Q}$.

Recommended Books

1. P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. I. N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
4. M. Artin, Algebra, Prentice Hall of India, New Delhi.
5. J. B. Fraleigh, A First course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

Additional Reference Books:

1. S. D. Adhikari , An Introduction to Commutative Algebra and Number theory, Narosa Publishing House.
2. T.W. Hungerford, Algebra, Springer.
3. D. Dummit, R. Foote, Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luthar, I.B.S. Passi, Algebra Vol. I and II, Narosa publication.
5. U. M. Swamy, A. V. S. N. Murthy, Algebra Abstract and Modern, Pearson.
6. Charles Lanski, Concepts Abstract Algebra, American Mathematical Society.
7. Sen, Ghosh and Mukhopadhyay, Topics in Abstract Algebra, Universities press.

T.Y.B.Sc.	Semester VI Theory
RJSUMAT602 Paper II Algebra VI	Course Outcome 6.2: 1. Normal groups, Quotient groups and Isomorphism theorems of groups. External direct product of groups 2. Ring theory ,Isomorphism theorem for Rings. 3. Ideals 4. Quotient Field. Learning Outcome : 1. Using Isomorphism to classify groups of orders ≤ 7 2. Understanding of Integral Domain and Division Ring. 3. Developing the concept of different type of Ideals in Rings 4. Learn to construct a quotient field

Paper 3: Topology of Metric Spaces-II
Course Code: RJSUMAT603

Unit 1: Compact sets (15 Lectures)

Definition of compact metric space using open cover, examples of compact sets in different metric spaces, sequentially compact metric space, Bolzano-Weierstrass property.

Properties of compact sets: A compact set is closed and bounded, (Converse is not true). Every infinite bounded subset of compact metric space has a limit point. A compact set has Bolzano-Weierstrass property. A compact set is sequentially compact.

A closed subset of a compact set is compact. Union and Intersection of Compact sets. Equivalent statements for compact sets in $\mathbb{R}^n$: Sequentially compactness property, Heine-Borel property, closed and boundedness property. Bolzano-Weierstrass property.

Unit 2: Continuous functions on metric spaces (15 Lectures)

Epsilon-delta definition of continuity at a point of a function from one metric space to another. Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples, Algebra of continuous real valued functions on a metric space. Continuity of composite continuous function. Continuous image of compact set is compact, uniform continuity in a metric space, definition and examples (emphasis on $\mathbb{R}$). Let (X, d) and (Y, d') be metric spaces and f: X → Y be continuous, where (X, d) is a compact metric space, then f: X→Y is uniformly continuous. Contraction mapping and fixed point theorem, Applications.

Unit 3: Connected sets: (15Lectures)

Separated sets- Definition and examples, disconnected sets, disconnected and connected metric spaces, connected subsets of a metric space, Connected subsets of $\mathbb{R}$. A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected. Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from X to {1, -1} is a constant function. Path connectedness in $\mathbb{R}^n$, definition and examples. A path connected subset of $\mathbb{R}^n$ is connected, convex sets are path connected. Connected components. An example of a connected subset of $\mathbb{R}^n$ which is not path connected.

References for Units I, II, III:

1. S. Kumaresan, Topology of Metric spaces.
2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi.

3. Robert Bartle and Donald R. Sherbert, Introduction to Real Analysis, Second Edition, John Wiley and Sons.
4. Ajit Kumar, S. Kumaresan, Basic course in Real Analysis, CRC press
5. R.R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi.

Other references:

1. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, New York.
2. W. A. Sutherland, Introduction to metric & topological spaces, Second Edition, Oxford.
3. T. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi.
4. P.K.Jain, K. Ahmed, Metric Spaces, Narosa, New Delhi.
5. W. Rudin, Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland.
6. D. Somasundaram, B. Choudhary, A first Course in Mathematical Analysis, Narosa, New Delhi.

T.Y.B.Sc.	Semester VI Theory
RJSUMAT603 Paper III Topology of Metric Spaces-II	Course Outcome 6.3: 1. To study compactness in metric spaces 2. To study compactness in Euclidean space 3. To learn important properties of compactness 4. To study continuous functions in metric spaces using $\varepsilon - \delta$ definition as well as in terms of sequence 5. Contraction mapping and fixed point theorem 6. connected metric spaces, path connectedness Learning Outcome : 1. Compact metric space and its properties 2. Continuous functions in metric spaces 3. Connected and disconnected subsets and their characterization

Paper 4: Number Theory and Its applications – II [Elective A]
Code: RJSUMAT604A

Unit 1: Quadratic Reciprocity (15 lectures)

Quadratic residues and Legendre Symbol, Gauss' Lemma, Theorem on Legendre Symbols $(\frac{-1}{p})$ and $(\frac{2}{p})$, Quadratic Reciprocity law and its applications, The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit 2: Continued Fractions (15 lectures)

Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions. Pell's equations and their solutions.

Unit 3: Arithmetic function and Special numbers (15 lectures)

Arithmetic functions of number theory: $\tau(n)$, $\sigma(n)$, $\sigma_s(n)$, $\omega(n)$, $\phi(n)$ and their properties, $\mu(n)$ and the Mobius inversion formula.

Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudoprimes, Carmichael numbers.

Recommended Books

1. Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc.
2. David M. Burton, An Introduction to the Theory of Numbers, Tata McGraw Hill Edition.
3. G. H. Hardy and E.M. Wright, An Introduction to the Theory of Numbers, Low priced edition, The English Language Book Society and Oxford University Press.
4. Neville Robins, Beginning Number Theory, Narosa Publications.
5. S.D. Adhikari, An introduction to Commutative Algebra and Number Theory, Narosa Publishing House.
6. N. Koblitz. A course in Number theory and Cryptography, Springer.
7. M. Artin, Algebra, Prentice Hall.
8. K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.
9. William Stallings, Cryptology and network security, Pearson Education.

10. T. Koshy, Elementary number theory with applications, 2nd edition, Academic Press.
11. A. Baker, A comprehensive course in number theory, Cambridge.

T.Y.B.Sc.	Semester VI Theory
RJSUMAT6034A Paper IV Number Theory and Its applications – II	Course Outcome 6.4A: 1. To understand use concepts and uses of finite continued fractions 2. To learn relation between irrational numbers and infinite continued fractions 3. Solving quadratic congruence using Legendre and Jacobi symbols 4. To understand number theoretic functions such as sigma, tau, Euler's 5. To learn special numbers such as Fermat numbers, Amicable numbers, perfect numbers and Messene numbers. Learning Outcome : 1. To understand that better approximations of irrational numbers can be done through continued fractions 2. To know Pell's equation and role of continued fractions in its solution. 3. To be able to solve quadratic congruences through the help of quadratic reciprocity law.

Paper IV: Numerical Analysis – II [Elective B]
Course Code: RJSUMAT604B

Unit I: Interpolation (15L)
Interpolating polynomials, Uniqueness of interpolating polynomials. Linear, Quadratic and Hillgher order interpolation. Lagranges Interpolation. Finite difference operators: Shift operator, forward, backward and central difference operator, Average operator and relation between them.
Difference table, Relation between difference and derivatives. Interpolating polynomials using finite differences Gregory-Newton forward difference interpolation, Gregory-Newton backward difference interpolation, Stirlings Interpolation. Results on interpolation error.

Unit II. Polynomial Approximations and Numerical Differentiation (15L)
Piecewise Interpolation: Linear, Quadratic and Cubic. Bivariate Interpolation: Lagranges Bivariate Interpolation, Newtons Bivariate Interpolation. Numerical differentiation: Numerical diferentiation based on Interpolation, Numerical differentiation based on finite differences (forward, backward and central), Numerical Partial differentiation.

Unit III. Numerical Integration (15L)
Numerical Integration based on Interpolation. Newton-Cotes Methods, Trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule. Determination of error term for all above methods.
Convergence of numerical integration: Necessary and sufficient condition (with proof). Composite integration methods; Trapezoidal rule, Simpson's rule.

Reference Books:
1. Kendall E, Atkinson, An Introduction to Numerical Analysis, Wiley.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain,, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. S.D. Conte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGraw Hill International Book Company.
4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.
5. Hildebrand F.B, .Introduction to Numerical Analysis, Dover Publication, NY.
6. Scarborough James B., Numerical Mathematical Analysis, Oxford University Press, New Delhi.

T.Y.B.Sc.	Semester VI Theory
RJSUMAT6034B Paper IV Numerical Analysis – II	Course Outcome 6.4B: 1. To learn Lagrange's Interpolation formula, Newton's forward and backward interpolating formulae 2. To learn Newton's and Lagrange's bivariate interpolation formulae 3. To learn Trapezoidal rule, Simpson's 1/3rd and Simpson's 3/8th rule and their convergence Learning Outcome : 1. To learn methods of interpolation 2. To understand numerical differentiation through polynomial approximations 3. To know methods of computing definite integrals

Course: Practicals based on RJSUMAT601 and RJSUMAT602
Course code: RJSUMATP601

List of suggested practicals based on RJSUMAT601:

1. Limit and continuity and sequence of complex numbers
2. Derivatives of complex functions , analyticity, harmonic functions
3. Elementary functions and Mobius transformation
4. Complex integration, Cauchy integral formula and Cauchy integral theorem
5. Taylor's series, Laurent series and singularities
6. calculation of residues and applications
7. Miscellaneous theoretical questions based on three units

List of suggested Practicals based on RJSUMAT602:

1. Normal Subgroups and quotient groups
2. Cayleys Theorem and external direct product of groups
3. Rings, Subrings, Ideals, Ring Homomorphism and Isomorphism
4. Prime Ideals and Maximal Ideals
5. Polynomial Rings
6. Fields
7. Miscellaneous Theoretical questions on Unit 1, 2 and 3

T.Y.B.Sc.	Semester VI Practical
RJSUMATP601 Based on RJSUMAT601 and RJSUMAT602	Course Outcome : 1. To solve problems based on limit, continuity and differentiability 2. Study of analytic, harmonic functions through examples 3. To write power series of given function and to identify its singularities 4. Quotient groups, External Direct products ,Ring, Subrings, Ideals Learning Outcome : 1. To be able to solve problems in complex analysis at elementary level 2. Understanding and solving problems based on Quotient groups, Rings,different type of ideals and able to differentiate among them.

Course: Practicals based on RJSUMAT603 and RJSUMAT604A/ RJSUMAT604B
Course code: RJSUMATP602

List of suggested practicals based on RJSUMAT603

1. Compact sets in various metric spaces

2. Compact sets in $\mathbb{R}^n$

3. Continuity in a metric space

4. Uniform continuity, contraction maps, fixed point theorem

5. Connectedness in metric spaces

6. Path connectedness

7. Miscellaneous theory questions on all units

List of suggested practicals based on RJSUMAT604A

1. Legendre Symbol, Gauss' Lemma, quadratic reciprocity law

2. Jacobi Symbol, quadratic congruences with prime and composite moduli

3. Finite and infinite continued fractions

4. Approximations and Pell's equations

5. Arithmetic functions of number theory

6. Special numbers

7. Miscellaneous

List of suggested practicals based on RJSUMAT604B:

1. Linear, Quadratic and Hillgher order interpolation, Interpolating polynomial by Lagranges Interpolation

2. Interpolating polynomial by Gregory-Newton forward and backward difference Interpolation and Stirling Interpolation.

3. Bivariate Interpolation: Lagranges Interpolation and Newtons Interpolation

4. Numerical differentiation: Finite differences (forward, backward and central), Numerical Partial differentiation

5. Numerical differentiation and Integration based on Interpolation

6. Numerical Integration: Trapezoidal rule, Simpsons 1/3rd rule, Simpsons 3/8th rule

7. Composite integration methods: Trapezoidal rule, Simpsons rule

T.Y.B.Sc.	Semester VI Practical
RJSUMATP602 Based on RJSUMAT603 and RJSUMAT604 (A/B)	Course Outcome 6.2: 1. Problems based on metric spaces 2. To learn continuity in metric spaces through examples 3. To study connetedness in matric spaces 4. To determine solvability of quadratic congruences 5. To generate infinite continued fraction 6. To implement Lagrange's and Newton's interpolation formulae 7. To implement Trapezoidal, Simpson's $1/3^{rd}$, Simpson's $3/8^{th}$ rule Learning Outcome : 1. Compactness in metric spaces

	2. Continuity in metric spaces 3. To know method for getting better approximations for irrational numbers 4. To learn properties of number theoretic functions 5. To know how to find approximate value missing data 6. To be able to find approximate value of definite integrals

Scheme of Examination

1. There will be theory examination of 100 marks for each of the courses RJSUMAT501, RJSUMAT502, RJSUMAT503, RJSUMAT504(A/B) and practical examination of 100 marks for each course RJSUMATP501 and RJSUMATP502 of semester V and theory examination of 100 marks for each of the courses RJSUMAT601, RJSUMAT602, RJSUMAT603, RJSUMAT604(A/B) and practical examination of 100 marks for each course RJSUMATP601 and RJSUMATP602 of semester VI.
2. Passing in theory and practical shall be separate.
3. Passing percentage is 40 percent.
4. In Theory Examination

   (i) There will be two Internal Assessments each of 20 marks and semester end examination of 60 marks for each of the courses RJSUMAT501, RJSUMAT502, RJSUMAT503, RJSUMAT504(A/B) of semester V and RJSUMAT601, RJSUMAT602, RJSUMAT603, RJSUMAT604(A/B) of semester VI.

   (ii) There will be combined passing (20+20+60=100 marks)

   (iii) Students have to compulsorily attempt Semester end examination and at least one internal assessment.

Internal Assessment : There will be two Internal Assessments each of 20 marks for each of the courses RJSUMAT501, RJSUMAT502, RJSUMAT503, RJSUMAT504(A/B) of semester V and RJSUMAT601, RJSUMAT602, RJSUMAT603, RJSUMAT604(A/B) of semester VI.

Internal Assessment I and II pattern:

(a) Objective type (five out of seven) (2X5=10 marks)

(b) Problems (two out of three) (5x2=10)

Semester End Theory Examinations: There will be a Semester end theory examination of 60 marks for each of the courses RJSUMAT501, RJSUMAT502, RJSUMAT503, RJSUMAT504(A/B) of semester V and RJSUMAT601, RJSUMAT602, RJSUMAT603, RJSUMAT604(A/B) of semester VI.

1. Duration: The examinations shall be of 2 Hours duration.

2. Theory Question Paper Pattern:
a) There shall be three questions Q1, Q2, Q3 each of 20 marks and each based on the units 1, 2, 3 respectively.
b) All the questions shall be compulsory. The questions Q1, Q2, Q3 shall have internal choices within the questions. Including the choices, the marks for each question shall be 40.
c) Each of the questions Q1, Q2, Q3 will be subdivided into two sub-questions as follows:

(i) Attempt any one out of two questions (each of 8 marks).
(ii) Attempt any two out of four questions (each of 6 marks)

Semester End Practical Examinations:
At the end of the Semesters V & VI Practical examinations of three hours duration and 100 marks shall be conducted for the courses RJSUMATP501, RJSUMATP502 of semester V and RJSUMATP601, RJSUMATP602 of semester VI.
In semester V, the Practical examinations for RJSUMATP501 and RJSUMATP502 will be held together.
In Semester VI, the Practical examinations for RJSUMATP601 and RJSUMATP602 will be held together.
Paper pattern: The question paper shall have two parts A and B.
Each part shall have two Sections.
Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. (8 x 2 = 16 Marks)
Section II Problems: Three questions based on each unit with internal choices. (8x 3 = 24 Marks)

Practical Course	Part A	Part B	Marks out of	duration
RJSUMATP501	Questions from RJSUMAT501	Questions from RJSUMAT502	80	3 hours
RJSUMATP502	Questions from RJSUMAT503	Questions from RJSUMAT504(A/B)	80	3 hours
RJSUMATP601	Questions from RJSUMAT601	Questions from RJSUMAT602	80	3 hours
RJSUMATP602	Questions from RJSUMAT603	Questions from RJSUMAT604(A/B)	80	3 hours

Marks for Journals:

For each course RJSUMAT501, RJSUMAT502, RJSUMAT503, RJSUMAT504, RJSUMAT601, RJSUMAT602, RJSUMAT603, RJSUMAT604:

Journals: 10 marks.

Each Practical of every course of Semester V and VI shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.