Hindi vidya Prachar Samiti's

Ramniranjan Jhunjhunwala College

of Arts, Science & Commerce

(Autonomous College)

Affiliated to

UNIVERSITY OF MUMBAI

Syllabus for the F.Y.B.Sc.

Program: B.Sc. Mathematics

Program Code: RJSUMAT

Choice based Credit System (CBCS)

With effect from the academic year 2018-19

SEMESTER I

Course Code	UNIT	TOPICS	Credits	L/Week
RJSUMAT101	**Paper 1: CALCULUS I**			
	I	Real Number System	2	3
	II	Sequences		
	III	Infinite Series		
RJSUMAT102	**Paper 2 : ALGEBRA I**			
	I	Integers and Divisibility	2	3
	II	Functions and equivalence Relation		
	III	Polynomials		
RJSUMATP101	**PRACTICALS**			
	-	Practicals based on RJSUMAT101 and RJSUMAT102	2	2

SEMESTER II

Paper 1 : CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/Week
RJSUMAT201	I	Graphs and Limit of functions	2	3
	II	Continuous functions and their applications		
	III	Differentiability and its applications		
Paper 2 : ALGEBRA II				
RJSUMAT202	I	System of Linear equations and Matrices	2	3
	II	Vector Spaces		
	III	Basis and Dimension of Vector Spaces		
PRACTICALS				
	-	Practicals based on RJSUMAT201	2	2

RJSUMATP201		and RJSUMAT202		

Semester - I

Paper 1: Calculus I (RJSUMAT101)

Unit 1: Real number system (15 Lectures)

Real number system $\mathbb{R}$ and order properties of $\mathbb{R}$, Absolute value and its properties, AM-GM inequality, Intervals and neighborhoods, Hausdorff property, Bounded sets, supremum, infimum and their properties, statement of L.U.B. axiom, Archimedean property and its applications, Density of rationals in $\mathbb{R}$.

Unit 2: Sequences (15 Lectures)

Sequences in $\mathbb{R}$, convergence and divergence of sequences, Boundedness of convergent sequence, Uniqueness of limit of a convergent sequence, Algebra of convergent sequences, Sandwich theorem, Monotone sequences, monotone convergence theorems and consequences.

Convergence of standard sequences like $\left(\frac{1}{1+na}\right)\ \forall a > 0$, $(b^n)\ \forall b \in [0,1]$, $\left(c^{\frac{1}{n}}\right)\ \forall\ c > 0$, $\left(n^{\frac{1}{n}}\right)$ and $\left(\left(1+\frac{1}{n}\right)^n\right)$. Subsequences, Cauchy sequence and examples. Every convergent sequence is a Cauchy sequence. Boundedness of a Cauchy sequence. A sequence is convergent iff every subsequence of it converges to the same limit.

Unit 3: Infinite Series (15 Lectures)

Infinite series of real numbers, convergent series, divergent series. Necessary condition for convergence of series. Algebra of convergent series, Cauchy's criterion, harmonic series, p-harmonic series, Comparison test, Limit comparison test, ratio test (without proof), root test (without proof) and examples, alternating series, Leibnitz test for alternating series, absolute convergence, conditional convergence.

Reference books:

1. Robert G. Bartle, Donald R. Sherbert, Introduction to Real Analysis, third edition, John Wiley & Sons, Inc.
2. R. R. Goldberg, Methods of real analysis, Indian Edition, Oxford and IBH publishing, New Delhi.
3. Tom M. Apostol, Calculus Vol.1, Second edition, John Wiley & Sons.
4. Sudhir R. Ghorpade, Balmohan V. Limaye, A Course in Calculus and Real Analysis, International edition, Springer.
5. Russell A. Gordon, Real Analysis A First Course, Second edition, Addison-Wesley.
6. S. C. Malik, Savita Arora, Mathematical Analysis, third edition, New Age International Publishers, India.
7. William Trench, Introduction to Real Analysis, Free hyperlinked edition.
8. D. Somasundaram, B. Choudhary, A First Course in Mathematical Analysis, corrected edition, Narosa Publishing House.
9. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press.
10. Charles G. Denlinger, Elements of Real Analysis, student edition, Jones & Bartlett.
11. M. Thamban Nair, Calculus of One Variable, student edition, Ane Books Pvt. Ltd.

F.Y.B.Sc.	Semester I Theory
RJSUMAT101 Paper 1 Calculus-I	Course Outcome 1.1: 1. Learning of real number system and some of its properties like, order property, Hausdorff property, Archimedean property 2. Understanding of bounded sets in $\mathbb{R}$ and supremum, infimum, LUB axiom and its consequences 3. To learn the concept of sequence of real numbers and its behavior using some results 4. To study subsequences and Cauchy sequence 5. Learning series of real numbers and its behavior using sequence of partial sums and some tests for convergence 6. Alternating series Learning Outcome : 1. Real number system and its algebraic properties,

	geometric properties 2. Bounded sets and their infimum and supremum 3. Sequence of real numbers and its convergence 4. Series of real numbers and its convergence

Paper II - ALGEBRA I (RJSUMAT102)

Prerequisites: Set Theory: Set, subset, union and intersection of sets, empty set, universal set, complement of a set, De Morgan's laws.

Unit I: Integers & divisibility (15 Lectures)

Statement of well-ordering property of non-negative integers, Principles of mathematical induction (first and second) as a consequence of well-ordering property.

Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a & b and that the g.c.d. can be expressed as $ma + nb$ for some $m, n \in \mathbb{Z}$, Euclidean algorithm.

Primes, Euclid's lemma, Fundamental theorem of arithmetic, The set of primes is infinite.

Congruences, definition and elementary properties, Euler's ϕ function, statements of Euler's theorem, Fermat's theorem and Wilson's theorem and their applications.

Unit II: Functions and Equivalence relations (15 Lectures)

Definition of a function, domain, co-domain and range of a function, composite functions, examples, Direct image f(A) and inverse image $f^{-1}(B)$ for a function f, injective, surjective, bijective functions, Composite of injective, surjective, bijective functions when defined, invertible functions, bijective functions are invertible and conversely. Examples of functions including constant, identity, projection, inclusion.

Binary operation, properties, examples.

Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.

Congruence is an equivalence relation on $\mathbb{Z}$, Residue classes and partition of $\mathbb{Z}$, addition and multiplication modulo n in $\mathbb{Z}_n$.

Unit III: Polynomials (15 Lectures)

Definition of polynomials over $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties.

Division algorithm in $F[X]$ (without proof), and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem.

A polynomial of degree n has at most n roots, Complex roots of a polynomial in $\mathbb{R}[X]$ occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree in $\mathbb{C}[X]$ has exactly n complex roots counted with multiplicity, Rational root theorem, simple consequences such as $\sqrt{p}$ is a irrational number where p is a prime number, Eisenstein's Criterion for irreducibility of a polynomial with integer coefficient (without proof), roots of unity, sum of all the roots of unity.

Reference Books

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.
2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford

Additional Reference Books

1. I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi
2. G. Birkhoff and S. Maclane, A Survey of Modern Algebra, Third Edition, MacMillan
3. N. S. Gopalkrishnan, University Algebra, New Age International Ltd
4. I .N. Herstein, Topics in Algebra, John Wiley
5. P. B. Bhattacharya , S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International
6. Kenneth Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition, Mathematics Series.
7. L. N. Childs, Concrete introduction to higher algebra, Third Edition, Springer

F.Y.B.Sc.	Semester I Theory
RJSUMAT102 Paper 2 Algebra-I	Course Outcome 1.2: 1. To learn concepts of integers such as gcd, lcm, primes, fundamental theorem of arithmetic

	2. To understand concepts of congruences, Fermat's theorem, Euler's theorem, Wilson's theorem and their role in large computations. 3. To learn basic terminologies of functions such as injective, surjective, bijective, inverse functions, composition of functions. 4. To understand binary operation and its various properties like commutative, associative, identity, inverse. 5. To understand concepts and of relations such as reflexive, symmetric, transitive, equivalence relations. 6. To understand terminologies of polynomials, finding roots of polynomials through rational root theorem. Learning Outcome : 1. To know properties of prime number 2. To understand mathematical induction as a proof technique 3. To understand basics of functions

Course: Practicals based on RJSUMAT102 and RJSUMAT102
Course Code: RJSUMATP101

List of practicals based on RJSUMAT101

1. Order properties, absolute value, AM-GM inequality, Hausdorff property,
2. Bounded sets, supremum and infimum, Archimedian property
3. Convergent sequences, divergent sequences, sandwich theorem
4. Monotone sequences, Cauchy sequences, Subsequences
5. Convergence of series
6. Alternating series, Absolute and conditional convergence
7. Miscellaneous theoretical questions based on three units

List of Practicals based on RJSUMAT102:

(1) Mathematical induction, Divisibility, GCD
(2) Primes and their properties, Congruences
(3) Functions

(4) Binary operations and Equivalence relations

(5) GCD of two polynomial, relation between roots and coefficients of polynomials, factorization.

(6) Rational root theorem, Eisenstein's Criterion

(7) Miscellaneous Theoretical Questions based on three units.

F.Y.B.Sc.	Semester I Practical
RJSUMATP101 Based on RJSUMAT101 and RJSUMAT102	Course Outcome: 1. To understand the method of expressing gcd as a linear combination 2. To simplify higher powers implementing the properties of congruences 3. To study real number system and its properties through examples 4. To study sequences and series of real numbers Learning Outcome : 1. To know methods of simplifying higher powers in integers through gcd, prime and congrunces 2. Real numbers and their algebraic and geometric properties 3. Sequence and series of real numbers

Semester - II

Paper 1: Calculus II (RJSUMAT201)

Unit 1: Graphs and limit of functions (15 Lectures)

Graphs of some standard functions such as $|x|$, e^x, $\ln x$, $\frac{1}{x}$, $\sin x$, $\cos x$, $\tan x$, $\sin\left(\frac{1}{x}\right)$, $x \sin\left(\frac{1}{x}\right)$, $x^2 \sin\left(\frac{1}{x}\right)$, $a x^2 + b x + c$ over suitable intervals in $\mathbb{R}$, conic sections.

$(\varepsilon - \delta)$ Definition and sequential definition of limit of a function at a point in $\mathbb{R}$. Evaluation of limit of simple functions using $(\varepsilon - \delta)$ definition, uniqueness of limit when it exists, Algebra of limits, Sandwich theorem for limits, one sided limits, non existence of limits, limit at infinity and infinite limits.

Unit 2: Continuity and its applications (15 Lectures)

Continuity of real valued functions with domain as intervals in $\mathbb{R}$, examples, continuity of functions at end points of interval, Sequential continuity, Algebra of continuous functions, continuity of composite functions. Discontinuous functions, examples of removable and essential discontinuities.

Sign preserving property of continuous function. Intermediate value theorem (statement only) and its applications. A continuous function defined on [a, b] is bounded and attains its bounds. Image of a closed interval under a continuous function is a closed interval.

Unit 3: Differentiability and its applications (15 Lectures)

Notion of differentiability with geometrical and physical interpretation, non differentiable functions, necessary condition for differentiability of real valued function, algebra of differentiable functions, derivative of inverse functions, chain rule.

Higher order derivatives, Leibnitz rule, implicit differentiation, Rolle's theorem, Lagrange's mean value theorems, Cauchy's mean value theorem, increasing and decreasing functions, extreme values, stationary points, first derivative test, second derivative test, point of inflection, convex and concave functions, L'Hopital's rule (statement only), Taylor's theorem with Lagrange's form of remainder (statement only).

Reference books:

1. Robert G. Bartle, Donald R. Sherbert, Introduction to Real Analysis, third edition, John Wiley & Sons, Inc.
2. R. R. Goldberg, Methods of real analysis, Indian Edition, Oxford and IBH publishing, New Delhi.
3. Tom M. Apostol, Calculus Vol.1, Second edition, John Wiley & Sons
4. Sudhir R. Ghorpade, Balmohan V. Limaye, A Course in Calculus and Real Analysis, International edition, Springer
5. Russell A. Gordon, Real Analysis A First Course, Second edition, Addison-Wesley
6. S. C. Malik, Savita Arora, Mathematical Analysis, third edition, New Age International Publishers, India.
7. William Trench, Introduction to Real Analysis, Free hyperlinked edition.
8. D. Somasundaram, B. Choudhary, A First Course in Mathematical Analysis, corrected edition, Narosa Publishing House
9. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press
10. Charles G. Denlinger, Elements of Real Analysis, student edition, Jones & Bartlett Publishers.
11. M. Thamban Nair, Calculus of One Variable, student edition, Ane Books Pvt. Ltd.

F.Y.B.Sc.	Semester II Theory
RJSUMAT201 Paper 1 Calculus -II	Course Outcome 2.1: 1. To know graphs of some standard functions 2. To learn advance results related to continuity of real functions 3. Finding of derivatives using Leibniz definition and some elementary results 4. To find stationary points, extreme values of functions using second derivative test 5. To Study Mean value theorems and their applications 6. Evaluating of indeterminate forms using L'Hopital's rule Learning Outcome : 1. Graphs of some standard functions and limits 2. Properties of continuous functions 3. Differentiation and its applications

Paper 2 - ALGEBRA -II(RJSUMAT202)

Prerequisites:

(1) Review of vectors in $\mathbb{R}^2$, $\mathbb{R}^3$ and as points, addition and scalar multiplication of vectors in terms of co-ordinates, dot-product structure, Scalar triple product, Length (norm) of a vector.

(2) Matrices

Unit I: System of Linear equations and Matrices (15 Lectures)

Definition of *n-tuples* of real numbers, sum of two *n-tuples* and scalar multiple of *n-tuples*. Parametric equation of lines and planes. System of homogeneous and non-homogeneous linear equations, the solution of system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for (m, n) = (1; 2); (1; 3); (2; 2); (2; 3); (3; 3).

Matrix units, elementary row operations, elementary matrices, invertible matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices. Row echelon form of matrices, rank of a matrix.

System of linear equations in matrix form, Gaussian elimination method to solve any system of linear equations and to find inverse of a matrix. System of m homogeneous linear equations in n unknowns has a non-trivial solution if m < n.

Unit II: Vector spaces (15 Lectures)

Definition of a real vector space, Examples such as $\mathbb{R}^n$; $\mathbb{R}[X]$; $M_{mxn}(\mathbb{R})$; space of all real valued functions on a non empty set.

Definition of a subspace of a vector space and examples such as: lines, planes passing through origin as sub-spaces of $\mathbb{R}^2$; $\mathbb{R}^3$ respectively; upper triangular matrices, diagonal matrices, symmetric matrices, skew-symmetric matrices as subspaces of $M_n(\mathbb{R})$ (n = 2, 3); $Pn(X) = \{a_{0+}\ a_1x + \ldots\ldots + a_nx^n\ /\ \ a_i \in \mathbb{R}\ \forall\ 0 \leq i \leq n\}$ as a subspace $\mathbb{R}$[X], the space of all solutions of the system of m homogeneous linear equations in n unknowns as a subspace of $\mathbb{R}^n$

Properties of a subspace such as necessary and sufficient condition for a non empty subset to be a subspace of a vector space, arbitrary intersection of sub-spaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is a subset of the other.

Finite linear combinations of vectors in a vector space; the linear span L(S) of a non-empty subset S of a vector space, S is a generating set for L(S); L(S) is a vector subspace of V.

Linearly independent/linearly dependent subsets of a vector space, a subset $\{v_1 , v_2 , \dots , v_k\}$ of a vector space is linearly dependent if and only if $\exists\ i \in \{1,2 , \dots , k\}$ such that v_i is a linear combination of the other vectors $v_j's$.

Unit III: Basis and Dimension of a finite dimensional vector space (15 L)

Basis of a vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any two basis of a vector space have the same number of elements, any set of n linearly independent vectors in an n dimensional vector space is a basis, any collection of n + 1 linearly independent vectors in an n-dimensional vector space is linearly dependent; extending any basis of a subspace W of a vector space V to a basis of the vector space V. If W_1, W_2 are two subspaces of a vector space V then $W_1 + W_2$ is a subspace of the vector space V of dimension $\dim(W_1) + \dim(W_1) - \dim(W_1 \cap W_2)$.

Row space, column space of an m x n matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, rank of a matrix , Invariance of rank upon elementary row or column operations.

Reference Books:

1. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer.
2. S. Kumaresan , Linear Algebra , Prentice Hall of India Pvt limited.
3. Gilbert Strang , Linear Algebra and its Applications, International Student Edition.
4. L. Smith, Linear Algebra, Springer Verlag.
5. A. Ramchandran Rao, P. Bhimashankaran; Linear Algebra Tata Mac Graw Hill.
6. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer Verlag, NewYork.
7. Sheldon Axler, Linear Algebra done right, Springer Verlag, New York.
8. Klaus Janich, Linear Algebra, Springer Verlag.
9. Otto Bretscher, Linear Algebra with Applications, Pearson Education.
10. Gareth Williams, Linear Algebra with Applications, Narosa Publication.
11. K.Hoffman and R. Kunze Linear Algebra, Tata MacGraw Hill, New Delhi.
12. H. Anton, Elementary Linear Algebra, Wiley publication.

F.Y.B.Sc.	Semester II Theory
RJSUMAT202 Paper 2 Algebra -II	Course Outcome 2.2: 1. System of linear equations in matrix form and methods to solve it. 2. Real Vector spaces and their properties , Subspaces 3. Basis and dimensions of a real vector space. Learning Outcome : 1. Learn Geometrical interpretation of simple system of equations. 2. Learning concepts like linear span, linearly dependent/ independent set. 3. Learning concepts like a Basis is a maximal linearly independent set and also minimal generating set of a real vector space.

Course: Practicals based on RJSUMAT201 and RJSUMAT202
Course Code: RJSUMATP201

List of practicals based on RJSUMAT201:

1. Drawing graphs of functions
2. Limits of functions, sandwich theorem, non existence of limits
3. Continuous functions ϵ-δ definition, sequential continuity
4. Applications of continuous functions
5. Leibnitz theorem, extreme values, convex and concave functions
6. Mean value theorems, Taylor's theorem
7. Miscellaneous theoretical questions based on three units

List of practicals based on RJSUMAT202:

(1) Solving homogeneous system of m equations in n unknowns and their geometrical interpretation for (m; n) =(1; 2); (1; 3); (2; 2); (2; 2); (3; 3), Row echelon form.

(2) Solving any m by n linear system of equations, elementary matrices and invertible matrices.

(3) Examples of vector spaces, Subspaces

(4) Linear span, Linearly dependent and Linearly Independent Set of a vector space

(5) Basis and Dimension of Vector Space.

(6) Row space, Column space, rank of a matrix

(7) Miscellaneous Theoretical Questions based on three units.

F.Y.B.Sc.	Semester II Practical
RJSUMATP201 Based on RJSUMAT201 and RJSUMAT202	Course Outcome: 1. To draw graph of functions 2. Continuity of function using $\varepsilon - \delta$ definition and sequence 3. Maxima and minima of functions 4. Mean value theorems 5. Solving system of equations 6. Examples of vector spaces Learning Outcome : 1. Continuous functions and their properties

	2. Higher order derivatives 3. Applications of derivatives 4. Learn Gaussian elimination method to solve the system. 5. Learn to check whether set is a vector space and find its dimension

Scheme of Examination

Internal Assessments: There will be two Internal Assessments each of 20 marks for each of the courses RJSUMAT101, RJSUMAT102 of Semester I and RJSUMAT201, RJSUMAT202 of semester II.

Internal Assessment I and II pattern:

(a) Objective type (five out of seven) (2X5=10 marks)

(b) Problems (two out of three) (5x2=10)

Semester End Theory Examinations: There will be a Semester end theory examination of 60 marks for each of the courses RJSUMAT101, RJSUMAT102 of Semester I and RJSUMAT201, RJSUMAT202 of semester II

1. Duration: The examinations shall be of 2 Hours duration.
2. Theory Question Paper Pattern:

a) There shall be Three questions Q1, Q2 , Q3 each of 20 marks and each based on the units I, II, III respectively.

b) All the questions shall be compulsory. The questions Q1, Q2, Q3 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.

c) Each of the questions Q1, Q2, Q3 will be subdivided into two sub-questions as follows:

(i) Attempt any one out of two questions (each of 8 marks).

(ii) Attempt any two out of four questions (each of 6 marks)

Semester End Practical Examinations:

At the end of the Semesters I & II Practical examinations of three hours duration and100 marks shall be conducted for the courses RJSUMATP101, RJSUMATP201.

In semester I, the Practical examinations for RJSUMAT101 and RJSUMAT102 will be held together.

In Semester II, the Practical examinations for RJSUMAT201 and RJSUMAT202 will be held together.

Paper pattern: The question paper shall have two parts A and B.

Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. (8 x 2 = 16 Marks)

Section II Problems: Three questions based on each unit with internal choices. (8x 3 = 24 Marks)

Practical Course	Part A	Part B	Marks out of	duration
RJSUMAT1P101	Questions from RJSUMAT101	Questions from RJSUMAT202	80	3 hours
RJSUMATP201	Questions from RJSUMAT201	Questions from RJSUMAT202	80	3 hours

Marks for Journals:
For each course RJSUMAT101, RJSUMAT102, RJSUMAT201 and RJSUMAT202: Journals: 10 marks.

Each Practical of every course of Semester I and II shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.