



**Hindi vidya Prachar Samiti's**  
**Ramniranjan Jhunjhunwala College**  
**of Arts, Science & Commerce**  
**(Autonomous College)**

**Affiliated to**  
**UNIVERSITY OF MUMBAI**  
**Syllabus for the S.Y.B.Sc.**  
**Program: B.Sc. Mathematics**  
**Program Code: RJSUMAT**

**Choice based Credit System (CBCS)**

**With effect from the academic year 2018-19**

**SEMESTER III**

Paper 1: MULTIVARIABLE CALCULUS				
Course Code	Unit	Topics	Credits	L/week
RJSUMAT301	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications of differentiation		
Paper 2: ALGEBRA III				
RJSUMAT302	I	Linear transformations and matrices	2	3
	II	Determinants		
	III	Inner product spaces		
Paper 3: DISCRETE MATHEMATICS				
RJSUMAT303	I	Permutations and recurrence relations	2	3
	II	Preliminary Counting		
	III	Advanced Counting		
PRACTICALS				
RJSUMATP301	--	Practicals based on RJSUMAT301, RJSUMAT302 and RJSUMAT303	3	5

**SEMESTER IV**

Course Code	Unit	Topics	Credits	L/week
RJSUMAT401	Paper 1: INTEGRAL CALCULUS			
	I	Riemann Integration	2	3
	II	Indefinite and Improper integrals		
	III	Applications and double integrals		
RJSUMAT402	Paper 2: ALGEBRA IV			
	I	Groups and subgroups	2	3
	II	Cyclic groups and cyclic subgroups		
	III	Lagrange's theorem and group homomorphisms		
RJSUMAT403	Paper 3: DIFFERENTIAL EQUATIONS			
	I	First order first degree differential equations	2	3
	II	Second order differential equations		
	III	Linear System ODEs and numerical methods		
PRACTICALS				
RJSUMATP401	--	Practicals based on RJSUMAT401, RJSUMAT402 and RJSUMAT403	3	5

## SEMESTER III

### Paper1: Multivariable Calculus

Course code: RJSUMAT301

#### Unit 1: Functions of Several Variables (15 Lectures)

Euclidean space  $\mathbb{R}^n$ , Euclidean norm function on  $\mathbb{R}^n$ , open ball and open sets in  $\mathbb{R}^n$ , sequences in  $\mathbb{R}^n$ , convergence of sequences and basic properties, subsequences (These concepts should be specifically discussed for  $\mathbb{R}^2$  and  $\mathbb{R}^3$ ).

Functions from  $\mathbb{R}^n$  to  $\mathbb{R}$  (scalar fields) and from  $\mathbb{R}^n$  to  $\mathbb{R}^m$  (vector fields), limits and continuity of scalar fields and vector fields, basic results on algebra of limits and continuity, nonexistence of limits, relation between continuity of vector field and its component functions.

Directional Derivatives and Partial derivatives of scalar fields, higher order partial derivatives, gradient of a scalar field, mean value theorem for derivatives of scalar fields.

#### Unit 2: Differentiation (15 Lectures)

Differentiability of a scalar field at a point of  $\mathbb{R}^n$  (in terms of linear transformation) and on open subsets of  $\mathbb{R}^n$ , the total derivative and its properties, uniqueness of total derivative of differentiable functions, differentiability of scalar field implies its continuity, necessary condition for differentiability, sufficient condition for differentiability, chain rule for derivatives of scalar fields, homogeneous functions and Euler's theorem, sufficient condition for equality of mixed partial derivatives (without proof).

#### Unit 3: Applications of Differentiation (15 Lectures)

Derivatives of vector fields, differentiability in terms of linear transformation, Jacobian matrix, differentiability of scalar field implies its continuity, chain rule for derivatives of vector fields (without proof).

Geometric properties of gradient of a scalar field, applications to geometry, level sets, tangent planes, Taylor's formula for functions of two variables (without proof), linear approximation, quadratic approximation, Hessian matrix, extreme values, saddle points, first derivative test, second partial derivative test (without proof), method of Lagrange's Multipliers.

**Reference Books:**

1. Tom M. Apostol, Calculus Vol. 2, second edition, John Wiley, India.
2. Jerrold E. Marsden, Anthony J. Tromba, Alan Weinstein, Basic Multivariable Calculus, Indian edition, Springer-Verlag.
3. Jerrold E. Marsden, Anthony J. Tromba, Vector Calculus, fifth edition, W.H. Freeman and Co, New York.
4. S.C. Malik, Savita Arora, Mathematical Analysis, third edition, New Age International Publishers, India.
5. D. Somasundaram, A Second Course in Mathematical Analysis, Narosa Publishing House, India.
6. Dennis G. Zill, Warren S. Wright, Calculus Early Transcendentals, fourth edition, Jones and Bartlett Publishers.
7. Sudhir R. Ghorpade, Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer.
8. Satish Shirali, Harkrishnan Lal Vasudeva, Multivariable Analysis, Springer.
9. William Trench, Introduction to Real Analysis, Free hyperlinked edition.

S.Y.B.Sc.	Semester III Theory
RJSUMAT301 Paper I Multivariable Calculus	<p>Course Outcome 3.1:</p> <ol style="list-style-type: none"><li>1. To extend the concept of limit and continuity for multivariable functions</li><li>2. To study directional derivatives, partial derivatives and Mean value theorem for derivatives of scalar fields</li><li>3. To define total derivative as a linear transformation and to discuss relations between directional derivatives, partial derivatives and total derivatives</li><li>4. To discuss chain rule for scalar fields, higher order and mixed partial derivatives</li><li>5. To understand differentiability of vector fields, special matrices viz. Jacobian and Hessian</li><li>6. To study methods of finding maxima and minima for functions of two variables</li></ol> <p>Learning Outcome :</p> <ol style="list-style-type: none"><li>1. Various definitions of derivatives of multivariable functions</li><li>2. Applications to find extreme values</li><li>3. Calculus for scalar and vector fields</li></ol>

**PAPER - II ALGEBRA III**  
**Course Code: RJSUMAT302**

**Unit I: Linear transformations and matrices (15 Lectures)**

Linear transformations: Kernel, Image of a linear transformation, Rank T, Nullity T, and properties such as: for a linear transformation T, kernel (T) is a subspace of the domain space of T and the image(T) is a subspace of the co-domain space of T. If V, W are real vector spaces with  $\{v_1, v_2, \dots, v_n\}$  a basis of V and  $\{w_1, w_2, \dots, w_n\}$  any vectors in W then there exists a unique linear transformation  $T: V \rightarrow W$  such that  $T(v_j) = w_j \forall 1 \leq j \leq n$ , Rank nullity theorem and examples. Linear isomorphisms, inverse of a linear isomorphism. Any n-dimensional real vector space is isomorphic to  $\mathbb{R}^n$ . Representation of linear maps by matrices and effect under a change of basis, examples.

Equivalence of rank of an  $m \times n$  matrix A and rank of the linear transformation  $L_A: \mathbb{R}^n \rightarrow \mathbb{R}^m (L_A(X) = AX)$ . The dimension of solution space of the system of linear equations  $AX = 0$  equals  $n - \text{rank}(A)$ . The solutions of non-homogeneous systems of linear equations represented by  $AX = B$ . Existence of a solution when  $\text{rank}(A) = \text{rank}(A; B)$

**Unit II: Determinants (15 Lectures)**

Definition of determinant as an n-linear skew-symmetric function from  $\mathbb{R}^n \times \mathbb{R}^n \times \dots \times \mathbb{R}^n \rightarrow \mathbb{R}$  such that determinant of  $(E^1, E^2, \dots, E^n)$  is 1, where  $E^j$  denotes the jth column of the  $n \times n$  identity matrix  $I_n$ . Determinant of a matrix as determinant of its column vectors (or row vectors).

Existence and uniqueness of determinant function via permutations, Computation of determinant of  $2 \times 2$ ,  $3 \times 3$  matrices, diagonal matrices, Basic results on determinants such as  $\det(A^t) = \det(A)$ ;  $\det(AB) = \det(A) \det(B)$ , Laplace expansion of a determinant, determinant of upper triangular and lower triangular matrices, Vandermonde determinant.

Linear dependence and independence of vectors in  $\mathbb{R}^n$  using determinants, the existence and uniqueness of the system  $AX = B$ , where  $A$  is an  $n \times n$  matrix with  $\det(A) \neq 0$ ,  $A^{-1} = \frac{1}{\det(A)} \text{adj}(A)$  for an invertible matrix  $A$ . Cramer's rule.

### Unit III: Inner Product Spaces (15 Lectures)

Dot product in  $\mathbb{R}^n$ . Definition of general inner product on a vector space over  $\mathbb{R}$ . Examples of inner product including the inner product  $\langle f, g \rangle = \int_{-\pi}^{\pi} f(t)g(t) dt$  on  $C[-\pi, \pi]$ , the space of continuous real valued functions on  $[-\pi, \pi]$ .

Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem and geometric applications in  $\mathbb{R}^2$ , Projections on a line, the projection being the closest approximation, orthogonal complements of a subspace, Orthogonal complements in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ . Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, Simple examples in  $\mathbb{R}^3$  and  $\mathbb{R}^4$ .

#### Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

#### Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.



5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
6. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer Verlag New York.
7. Sheldon Axler, Linear Algebra done right, Springer Verlag.
8. Klaus Janich, Linear Algebra, Springer Verlag.
9. Otto Bretscher, Linear Algebra with Applications, Pearson Education.
10. Gareth Williams, Linear Algebra with Applications, Narosa Publication.

S.Y.B.Sc.	Semester III Theory
RJSUMAT302	Course Outcome 3.2:
Paper II	1. kernel and image of linear transformation, linear isomorphism.
ALGEBRA III	2. Determinants using a bilinear map and their properties.
	3. Inner product spaces, Cauchy-Schwartz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem.
	Learning Outcome :
	1. Learn to find rank and nullity of a linear transformation.
	2. Learn the existence and uniqueness of system $Ax=b$ using determinant and its applications.
	3. Learn to find corresponding orthogonal/orthonormal set from a linearly independent set in a vector space.

### Paper III: Discrete Mathematics

Course Code: RJSUMAT303

#### Unit I: Permutations and Recurrence relation (15 Lectures)

1. Permutation of objects,  $S_n$ , composition of permutations, definition of cycles, transposition, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, even and odd permutations, definition of  $A_n$ , signature of a permutation, cardinality of  $S_n$  and  $A_n$ . Order of elements of  $S_n$ .
2. Recurrence Relations, definition of homogeneous, non-homogeneous, linear, nonlinear recurrence relation, obtaining recurrence relation in counting problems, solving homogeneous as well as non-homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method.

#### Unit II: Preliminary Counting (15 Lectures)

1. Finite and infinite sets, countable and uncountable sets examples such as  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{N} \times \mathbb{N}$ ,  $\mathbb{Q}$ ,  $(0, 1)$ ,  $\mathbb{R}$ .
2. Addition and multiplication Principle, counting sets of pairs, two ways counting.
3. Stirling numbers of second kind. Simple recursion formulae satisfied by  $S(n, k)$  for  $k = 1, 2, \dots, n$ .
4. Pigeonhole principle and its strong form, its applications to geometry, monotonic sequences etc.

#### Unit III: Advanced Counting (15 Lectures)

1. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.
  - $\sum_{k=0}^r \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}$
  - $\sum_{k=0}^n \binom{i}{r} = \binom{n+1}{r+1}$
  - $\sum_{i=0}^k \binom{k}{i}^2 = \binom{2k}{k}$
  - $\sum_{i=0}^n \binom{n}{i} = 2^n$
2. Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
3. Non-negative and positive solutions of equation  $x_1 + x_2 + \dots + x_k = n$
4. Principle of inclusion and exclusion, its applications, derangements, explicit formula for  $d_n$ , deriving formula for Euler's function  $\phi(n)$ .

**Recommended Books:**

1. Norman Biggs, Discrete Mathematics, Oxford University Press.
2. Richard Brualdi, Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy, Combinatorics-Theory and Applications, Affiliated East West Press.
4. S. S. Sane, Combinatorial Techniques, Hindustan Book Agency.
5. K. Rosen, Discrete Mathematics and its Applications, Tata McGraw Hills.
6. Schaum's outline series, Discrete mathematics.
7. Applied Combinatorics, Allen Tucker, John Wiley and Sons.
8. R. A. Beeler, How to count, Springer.

S.Y.B.Sc.	Semester IV Theory
RJSUMAT303 Paper III Discrete Mathematics	<p>Course Outcome 3.3:</p> <ol style="list-style-type: none"> <li>1. To know elementary definitions in <math>S_n</math> like transpositions, signature of a permutation, even and odd permutations</li> <li>2. To learn methods of solving first order and second order recurrence relations</li> <li>3. To learn cardinalities of sets and basic principles of counting</li> <li>4. To study Stirling numbers, Pigeonhole Principle</li> <li>5. To study multinomial theorem, inclusion-exclusion theorem and their applications</li> </ol> <p>Learning Outcome :</p> <ol style="list-style-type: none"> <li>1. To understand elementary calculations in <math>S_n</math></li> <li>2. To understand modeling problems through recurrence relations</li> <li>3. To know classification sets through their cardinalities</li> <li>4. To know basic and advanced techniques of counting</li> </ol>

**Course: Practical based on RJSUMAT301/ RJSUMAT302/ RJSUMAT303**

**Course Code: RJSUMATP301**

**List of suggested practicals based on RJSUMAT301:**

1. Open sets in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ , sequences in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ , limits and continuity of scalar fields and vector fields, nonexistence of limits of scalar fields
2. Directional derivatives, partial derivatives and Mean value theorem
3. Total derivative of scalar fields, chain rules, Euler's theorem for homogeneous functions
4. Total derivative of vector fields, Jacobian matrix, chain rule for derivative of vector fields
5. Level sets, tangent planes, linear and quadratic approximations, Hessian matrix
6. Extreme values, saddle points and method of Lagrange's multipliers
7. Miscellaneous Theoretical Questions based on three units

**Suggested Practical for RJSUMAT302:**

1. Linear Transformation
2. System of linear equations
3. Determinants
4. Finding inverse of  $n \times n$  matrices using adjoint ( $n \leq 3$ )
5. Inner product spaces, examples. Orthogonal complements in  $\mathbb{R}^2$  and  $\mathbb{R}^3$
6. Gram-Schmidt method
7. Miscellaneous Theoretical Questions based on full paper

**List of suggested practicals based on RJSUMAT303**

1. Various calculations involving permutations
2. Formulating and solving recurrence relations
3. Cardinality of sets, problems based on counting principles, two way counting
4. Stirling number of second kind and pigeon hole principles
5. Multinomial theorem, permutations and combinations of multi-sets
6. Inclusion-exclusion principle, derangements and Euler's phi function
7. Miscellaneous theory questions from all units

S.Y.B.Sc.	Semester III Practical
RJSUMATP301 Based on RJSUMAT301, RJSUMAT302, RJSUMAT303	<p>Course Outcome:</p> <ol style="list-style-type: none"> <li>1. Problems based on limit and continuity of multivariable functions</li> <li>2. Finding derivatives of multivariable functions</li> <li>3. Linear transformations, Determinants and Inner product spaces</li> <li>4. To do various computations in <math>S_n</math>, formulating and solving recurrence relations</li> <li>5. To solve various problems based elementary and advanced counting</li> </ol> <p>Learning Outcome :</p> <ol style="list-style-type: none"> <li>1. To learn basic concepts of calculus for scalar and vector fields</li> <li>2. Finding kernel and Image of Linear transformations, using Cramer's rule and gram schmidt's orthogonalisation process.</li> <li>3. To understand basic calculations in permutations and counting techniques</li> </ol>

## SEMESTER IV

### Paper I: Integral Calculus

Course code: RJSUMAT401

#### Unit I: Riemann Integration (15 Lectures)

Approximation of area, upper and lower Riemann sums and their properties, upper and lower integrals, definition of Riemann integral on a closed and bounded interval, Riemann criterion for integrability, examples of non Riemann integrable functions.

Results:

- (i) If  $f: [a, b] \rightarrow \mathbb{R}$  is continuous, then  $f$  is Riemann integrable on  $[a, b]$ .
- (ii) If  $f: [a, b] \rightarrow \mathbb{R}$  is monotone, then  $f$  is Riemann integrable on  $[a, b]$ .
- (iii) If  $f: [a, b] \rightarrow \mathbb{R}$  is bounded with finite number of discontinuities, then  $f$  is Riemann integrable on  $[a, b]$ .

Properties:

- (i) If  $a < c < b$ , then  $f \in R[a, b]$  if and only if  $f \in R[a, c]$  and  $f \in R[c, b]$ , further  $\int_a^b f = \int_a^c f + \int_c^b f$ .
- (ii) If  $f, g$  are Riemann integrable on  $[a, b]$ , then  $f + g$  is Riemann integrable on  $[a, b]$  and  $\lambda f$  is Riemann integrable on  $[a, b]$  for some  $\lambda \in \mathbb{R}$ . Further  $\int_a^b (f + g) = \int_a^b f + \int_a^b g$  and  $\int_a^b \lambda f = \lambda \int_a^b f$ .
- (iii) If  $f$  is Riemann integrable on  $[a, b]$ , then  $|f|$  is Riemann integrable on  $[a, b]$  and  $\left| \int_a^b f \right| \leq \int_a^b |f|$ , but not conversely.
- (iv) If  $f: [a, b] \rightarrow \mathbb{R}$  is Riemann integrable and  $f \geq 0$ , then  $\int_a^b f \geq 0$ .

Definition of Indefinite integral and its continuity, Fundamental theorems of calculus, mean value theorems, integration by parts, Leibnitz rule.

## Unit II: Applications and improper integrals (15 Lectures)

Applications of definite integrals: area between curves, volume by slicing, volumes of solids of revolution, length of a plane curve, area of surfaces of revolution.

Improper integrals of 1<sup>st</sup> Kind and 2<sup>nd</sup> Kind, absolute convergence of improper integrals, comparison tests, Abel's test (statement only) and Dirichlet's test (statement only).

Beta and gamma functions and their properties, relationship between beta and gamma functions, duplication formula.

## Unit III: Multiple Integrals (15 Lectures)

Definition of double (resp: triple) integral of a function and bounded on a rectangle (resp: box). Geometric interpretation as area and volume. Basic properties of double and triple integrals such as:

(i) Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions.

(ii) Integrability of continuous functions.

(iii) Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains. Change of variables formula (without proof). Polar, cylindrical and spherical coordinates, and integration using these coordinates. Differentiation under the integral sign.

(iv) Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals.

### Reference Books:

1. William Trench, Introduction to Real Analysis, Free hyperlinked edition.
2. S.C. Malik, Savita Arora, Mathematical Analysis, third edition, New Age International Publishers, India.

3. Sudhir R. Ghorpade, Balmohan V. Limaye, A Course in Calculus and Real Analysis, International edition, Springer.
4. H.S. Dami, Integral Calculus, New Age International Publishers, India.
5. D. Somasundaram, B. Choudhary, A First Course in Mathematical Analysis, corrected edition, Narosa Publishing House.
6. Shanti Narayan, P.K. Mittal, Integral Calculus, S. Chand Publishers.
7. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press.
8. Charles G. Denlinger, Elements of Real Analysis, student edition, Jones & Bartlett Publisher.
9. M. Thamban Nair, Calculus of One Variable, student edition, Ane Books Pvt. Ltd.
10. Tom M. Apostol, Calculus Vol.2, Second edition, Wiley-India.
11. Jerrold E. Marsden, Anthony J. Tromba, Alan Weinstein, Basic Multivariable Calculus, Indian edition, Springer-India.

S.Y.B.Sc.	Semester IV Theory
RJSUMAT401 Paper I Integral Calculus	<p>Course Outcome 4.1:</p> <ol style="list-style-type: none"> <li>1. Approximating area using Riemann upper and lower sums and finding Riemann integral of functions using definition</li> <li>2. To characterize class of Riemann integrable functions and some basic results</li> <li>3. To Study Fundamental theorems of calculus, Mean value theorems for integrals and integration by parts</li> <li>4. Applications of definite integrals to find area, volume and length of the curve</li> <li>5. To analyze behavior of two improper integrals using some test</li> <li>6. To define double integrals in Cartesian and polar forms</li> </ol>



	<p>and its applications</p> <p>Learning Outcome :</p> <ol style="list-style-type: none"><li>1. To know actual definition of integration</li><li>2. Relations between derivative and integral</li><li>3. Study of improper integrals and their applications</li><li>4. double, triple integrals and their applications</li></ol>
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**PAPER - II ALGEBRA IV**  
**Course Code: RJSUMAT402**

**Unit I: Groups and Subgroups (15 Lectures)**

(a) Definition of a group, Abelian group, Order of a group, finite groups, infinite groups.

Examples of groups including:

1.  $\mathbb{Z}, \mathbb{R}, \mathbb{C}$  under addition.
2.  $Q^* (= \mathbb{Q} \setminus \{0\})$ ;  $\mathbb{R}^* (= \mathbb{R} \setminus \{0\})$ ,  $Q^+$  (= positive rational numbers),  $\mathbb{C}^*$  under multiplication.
3.  $\mathbb{Z}_n$  (=the group of residue classes modulo  $n$ ) under addition.
4.  $U(n)$  (= the group of prime residue classes modulo  $n$ ) under multiplication
5.  $S_n$  (=the group of all permutations of  $\{1, 2, \dots, n\}$ ).
6. Klein 4-group.
7. The group of symmetries of a plane figure. The Dihedral group  $D_n$  (= the group of symmetries of a regular polygon of  $n$  sides (for  $n=3,4$ )).
8. Quaternion group.
9.  $M_{m \times n}(\mathbb{R})$  (=the group of all  $m \times n$ -matrices with real entries) under addition of matrices and  $GL_n(\mathbb{R})$ ,  $SL_n(\mathbb{R})$
10. Examples such as  $S^1$  as subgroup of  $\mathbb{C}$ .

(b) Properties such as

- 1) In a group  $(G, .)$  the following indices rules are true for all integers  $n, m$ 
  - i)  $a^n a^m = a^{n+m}$  for all  $a$  in  $G$
  - ii)  $(a^n)^m = a^{nm}$  for all  $a$  in  $G$
  - iii)  $(ab)^n = a^n b^n$  for all  $ab$  in  $G$  whenever  $ab = ba$

2) In a group  $(G, .)$  the following are true:

- i) The identity element  $e$  of  $G$  is unique.
- ii) The inverse of every element in  $G$  is unique.
- iii)  $(a^{-1})^{-1} = a$  for all  $a$  in  $G$ .
- iv)  $(ab)^{-1} = b^{-1} a^{-1}$  for all  $a, b$  in  $G$ .
- v) If  $a^2 = e$  for every  $a$  in  $G$  then  $(G, .)$  is an abelian group.
- vi) If  $(ab)^2 = a^2 b^2$  for every  $a, b$  in  $G$  then  $(G, .)$  is an abelian group.
- vii)  $(aba^{-1})^n = ab^n a^{-1}$  for every  $a, b$  in  $G$  and for every integer  $n$ .
- viii)  $(\mathbb{Z}_n^*, .)$  is a group if and only if  $n$  is a prime.

- 3) Properties of order of an element such as: ( $n$  and  $m$  are integers.)
- i) If  $o(a) = n$  then  $a^m = e$  if and only if  $n|m$ .
  - ii) If  $o(a) = nm$  then  $o(a^n) = m$ .
  - iii) If  $o(a) = n$  then  $o(a^m) = \frac{n}{(n,m)}$ , where  $(n, m)$  is the GCD of  $n$  and  $m$ .
  - iv)  $o(aba^{-1}) = o(b)$  and  $o(ab) = o(ba)$ .
  - v) If  $o(a) = m$  and  $o(b) = n$ ;  $ab = ba$ ;  $(n, m) = 1$  then  $o(ab) = nm$ .

(c) Subgroups

- i) Definition, necessary and sufficient condition for a non-empty subset of a group to be a subgroup.
- ii) The center  $Z(G)$  of a group is a subgroup.
- iii) Intersection of two (or a family of) subgroups is a subgroup.
- iv) Union of two subgroups is not a subgroup in general. Union of two subgroups is a subgroup if and only if one is contained in the other.
- v) If  $H$  and  $K$  are subgroups of a group  $G$  then  $HK$  is a subgroup of  $G$  if and only if  $HK = KH$ .

**Unit II: Cyclic groups and cyclic subgroups (15 Lectures)**

(a) Cyclic subgroup of a group, cyclic groups, (examples including  $\mathbb{Z}$ ,  $\mathbb{Z}_n$  and  $\mu_n$ ).

(b) Properties such as:

- (i) Every cyclic group is abelian.
- (ii) Finite cyclic groups, infinite cyclic groups and their generators.
- (iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group.
- (iv) Subgroup of a cyclic group is cyclic.
- (v) In a finite group  $G$ ,  $G = \langle a \rangle$  if and only if  $o(G) = o(a)$ .
- (vi) If  $G = \langle a \rangle$  and  $o(a) = n$  then  $G = \langle a^m \rangle$  if and only if  $(n, m) = 1$ .
- (vii) If  $G$  is a cyclic group of order  $p^n$  and  $H < G$ ,  $K < G$  then prove that either  $H \subseteq K$  or  $K \subseteq H$ .

**Unit III: Lagrange's Theorem and Group homomorphism (15 Lectures)**

(a) Definition of Coset and properties such as :

- 1) IF  $H$  is a subgroup of a group  $G$  and  $x \in G$  then
  - (i)  $xH = H$  if and only if  $x \in H$ .

- (ii)  $Hx = H$  if and only if  $x \in H$ :
- 2) If  $H$  is a subgroup of a group  $G$  and  $x, y \in G$  then
  - (i)  $xH = yH$  if and only if  $x^{-1}y \in H$
  - (ii)  $Hx = Hy$  if and only if  $xy^{-1} \in H$ .
- 3) Lagrange's theorem and consequences such as Fermat's little theorem, Euler's theorem and if a group  $G$  has no nontrivial subgroups then order of  $G$  is a prime and  $G$  is Cyclic.
- (b) Group homomorphisms and isomorphisms, automorphisms
  - i) Definition.
  - ii) Kernel and image of a group homomorphism.
  - iii) Examples including inner automorphism.

Properties such as:

- (1)  $f : G \rightarrow G'$  is a group homomorphism then  $\ker f \leq G$ .
- (2)  $f : G \rightarrow G'$  is a group homomorphism then  $\ker f = \{e\}$  if and only if  $f$  is 1-1.
- (3)  $f : G \rightarrow G'$  is a group isomorphism then
  - (i)  $G$  is abelian if and only if  $G'$  is abelian.
  - (ii)  $G$  is cyclic if and only if  $G'$  is cyclic.

### Recommended Books:

1. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
2. N.S. Gopalkrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. P.B. Bhattacharya, S.K. Jain, S. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi.
5. I.N. Herstein, Topics in Algebra, Second edition, Wiley Eastern Limited.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

### Additional Reference Books:

1. S. D. Adhikari. An introduction to Commutative Algebra and Number theory,

Narosa Publishing House.

2. T. W. Hungerford, Algebra, Springer.

3. D. Dummit, R. Foote, Abstract Algebra, John Wiley & Sons, Inc.

4. I.S. Luther, I.B.S. Passi, Algebra - Vol. I and II, Narosa Publishing House.

S.Y.B.Sc.	Semester IV Theory
RJSUMAT402  Paper II  ALGEBRA IV	<p>Course Outcome 4.2:</p> <ol style="list-style-type: none"><li>1. Groups and its properties, subgroups.</li><li>2. Cyclic groups and subgroups.</li><li>3. Homomorphism and Isomorphism of groups.</li></ol> <p>Learning Outcome :</p> <ol style="list-style-type: none"><li>1. Detailed study of Group theory with examples.</li><li>2. Learn to find generators of a cyclic group.</li><li>3. Learning concepts like kernel, Image and properties of group Homomorphism</li></ol>

### Paper III: Differential Equations

Course Code: RJSUMAT403

#### Unit I: First order first degree differential equations (15 Lectures)

1. Definitions of: Differential Equation, Order and Degree of a differential Equation, Ordinary Differential Equation (ODE), Linear ODE, non-linear ODE.
2. Definition of Lipschitz function, examples. Existence and Uniqueness Theorem for the differential equation  $y' = f(x; y)$ ;  $y(x_0) = y_0$  where  $f(x; y)$  is a continuous function satisfying Lipschitz condition  $|f(x, y_1) - f(x, y_2)| \leq K|y_1 - y_2|$  on the strip  $a \leq x \leq b$  &  $y \in \mathbb{R}$  (without proof). Examples based on verifying the conditions of existence and uniqueness theorem.
3. Review of solution of homogeneous and non-homogeneous linear differential equations of first order and first degree. Exact Equations: General Solution of Exact equations of first order and first degree, Necessary and sufficient condition for  $Mdx + Ndy = 0$  to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non-exact equations.
4. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.

#### Unit II: Second order Linear Differential equations (15 Lectures)

1. Existence and Uniqueness Theorem for the solutions of a second order linear ODE:  $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$  with initial conditions  $y(x_0) = y_0$  &  $y'(x_0) = y_1$  where  $P(x)$ ,  $Q(x)$ ,  $R(x)$  are continuous functions on  $[a, b]$  (without proof). Examples based on verifying the conditions of existence and uniqueness theorem.
2. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-

homogeneous second order equation. Complementary functions and particular integrals.

3. The homogeneous equation with constant coefficients, auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.

4. The Cauchy-Euler equation.

### **Unit III: Non-homogeneous Second Order Linear ODE and Oscillation Theory (15 Lectures)**

1. The method of undetermined coefficients.

2. The method of variation of parameters.

3. Finiteness of the number of zeros in a closed and bounded interval of a solution of a second order homogeneous linear differential equation. Sturm's Separation Theorem.

4. Reduction of a second order equation to its normal form. Sturm's Comparison Theorem. Examples: Distribution of zeros of the Bessel's equation  $x^2y'' + xy' + (x^2 - p^2)y = 0$  and the Airy's equation  $y'' + xy = 0$ .

### **Reference books:**

1. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.

2. E. A. Codington, An introduction to ordinary differential equations, Dover Books.

3. S. L. Ross, Differential equations, 3<sup>rd</sup> edition, Wiley India Edition.

4. D. G. Zill, A first course in differential equations with modeling applications, 10<sup>th</sup> edition, Cengage Learning.

S.Y.B.Sc.	Semester IV Theory
RJSUMAT403 Paper III Differential Equations	<p>Course Outcome 4.3:</p> <ol style="list-style-type: none"><li>1. To study classification of differential equations, linear and non linear differential equations, homogeneous, non homogeneous , exact, Bernoulli's differential equations</li><li>2. To learn some applications of first order differential equations</li><li>3. To learn methods of solving second order differential equations with constant coefficients</li><li>4. To study methods of solving non homogeneous second order differential equations: Variation of parameters, method of undetermined coefficients</li><li>5. To understand Sturm's separation theorem, Sturm's comparison theorem</li></ol> <p>Learning Outcome :</p> <ol style="list-style-type: none"><li>1. To know different types of first order ODE and its applications,</li><li>2. To learn second order ODE, Wronskian, linear independence of solutions</li><li>3. To understand homogeneous second order ODE, complementary function and particular integral</li><li>4. To understand Qualitative properties of solutions</li></ol>



**Course: Practical based on RJSUMAT401/ RJSUMAT402/ RJSUMAT403**

**Course Code: RJSUMATP401**

**List of Practicals based on RJSUMAT401**

1. Calculation of upper and lower sums, Problems based on definition of Riemann integral.
2. Properties of Riemann integral, Non Riemann integrable functions.
3. Fundamental theorems of Calculus, Mean value theorems, integration by parts, Leibnitz rule.
4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, beta and gamma functions.
5. Finding area, volume, length.
6. Double integrals.
7. Miscellaneous theoretical questions based on three units.

**List of suggested practicals based on RJSUMAT402:**

1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle, and square.
3. Subgroups.
4. Cyclic groups, cyclic subgroups, Finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.
6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full paper.

**List of suggested practicals based on RJSUMAT403:**

1. Solving exact and non-exact ODEs
2. Linear and reducible to linear equations, applications to ODEs

3. Wronskian, Finding another linearly independent solution from known solution
4. Finding general solution of homogeneous second order differential equations with constant coefficients
5. Method of undetermined coefficients
6. Method of variation of parameters, Qualitative properties of solutions
7. Miscellaneous theoretical questions from all units

S.Y.B.Sc.	Semester IV Practical
RJSUMATP401 Based on RJSUMAT401, RJSUMAT402, RJSUMAT403	<p>Course Outcome:</p> <ol style="list-style-type: none"> <li>1. To apply definition of integration to find integral of certain functions</li> <li>2. To find area, volume and length of the curve using definite integral</li> <li>3. Applications of multiple integration</li> <li>4. Groups, Cyclic groups, Cosets and Homomorphism of groups</li> <li>5. To implement various methods to solve first order and second differential equations</li> </ol> <p>Learning Outcome :</p> <ol style="list-style-type: none"> <li>1. To understand the concept of integration thoroughly</li> <li>2. Exhaustive problems based on Group Theory</li> <li>3. Understanding of various tools for solving differential equations</li> </ol>

### Scheme of Examination

1. There will be theory examination of 100 marks for each of the courses RJSUMAT301, RJSUMAT302, RJSUMAT303 and practical examination of 150 marks for course RJSUMATP301 of semester III and theory examination of 100 marks for each of the courses RJSUMAT401, RJSUMAT402, RJSUMAT403 and practical examination of 150 marks for course RJSUMATP401 of semester IV. Passing in theory and practical shall be separate.
2. Passing percentage is 40 percent.
3. In Theory Examination
  - (i) There will be two Internal Assessments each of 20 marks and semester end examination of 60 marks for each of the courses RJSUMAT301, RJSUMAT302, RJSUMAT303 of semester III and RJSUMAT401, RJSUMAT402, RJSUMAT403 of semester IV.
  - (ii) There will be combined passing ( $20+20+60=100$  marks)
  - (iii) Students have to compulsorily attempt Semester end examination and at least one Internal Assessment.

**Internal Assessments :** There will be two Internal Assessment each of 20 marks for each of the courses RJSUMAT301, RJSUMAT302, RJSUMAT303 of semester III and RJSUMAT401, RJSUMAT402, RJSUMAT403 of semester IV.

**Internal Assessment I and II pattern:**

- (a) Objective type (five out of seven) ( $2 \times 5 = 10$  marks)
- (b) Problems (two out of three) ( $5 \times 2 = 10$ )

**Semester End Theory Examinations:** There will be a Semester end theory examination of 60 marks for each of the courses RJSUMAT301, RJSUMAT302, RJSUMAT303 of semester III and RJSUMAT401, RJSUMAT402, RJSUMAT403 of semester IV.

1. Duration: The examinations shall be of 2 Hours duration.
2. Theory Question Paper Pattern:

- a) There shall be three questions Q1, Q2, Q3 each of 20 marks and each based on the units 1, 2, 3 respectively.
- b) All the questions shall be compulsory. The questions Q1, Q2, Q3 shall have internal choices within the questions. Including the choices, the marks for each question shall be 40.
- c) Each of the questions Q1, Q2, Q3 will be subdivided into two sub-questions as follows :
- Attempt any one out of two questions (each of 8 marks)
  - Attempt any two out of four questions (each of 6 marks)

### Semester End Practical Examinations:

At the end of the Semesters III & IV Practical examinations of three hours duration and 150 marks shall be conducted for the courses RJSUMATP301, RJSUMATP401 of semester III and IV respectively.

**Paper pattern:** The question paper shall have three parts A and B.

Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. (8 x 2 = 16 Marks)

Section II Problems: Three questions based on each unit with internal choices. (3x 8 = 24 Marks)

Practical Course	Part A	Part B	Part C	Marks out of	duration
RJSUMATP301	Questions from RJSUMAT301	Questions from RJSUMAT302	Questions from RJSUMAT303	120	3 hours
RJSUMATP401	Questions from RJSUMAT401	Questions from RJSUMAT402	Questions from RJSUMAT403	120	3 hours

**Marks for Journals:**

For each course RJSUMAT301, RJSUMAT302, RJSUMAT303, RJSUMAT401, RJSUMAT402, RJSUMAT403:

Journals: 10 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.